Curve Sketching

Sketch the following functions, showing all work needed to sketch each curve.
Check answers using a graphing calculator. (10 each)

1. $f(x) = \dfrac{-5x^2 + 3x}{2x^2 - 5}$

2. $y = \dfrac{1}{3 + x^2}$

Deriv. of Exp. and Log Functions

1. Find the derivative of the following functions. (3 each)

   a. $y = \dfrac{2^x}{e^x}$

   b. $f(x) = 2x\ln(x^2 + 5)$

   c. $g(x) = \dfrac{\ln x}{e^{x^2+2}}$

2. If $s(t) = \ln(3t^2 + t)$ find the slope of the function at $t = 2$. (3 marks)

3. Find $\dfrac{dy}{dx}$ for the function $xy^2 + x\ln x = 4y$ for $x > 0$. (3 marks)

4. Graph the function $y = e^{-x^2}$.

5. Use logarithmic differentiation to find $\frac{dy}{dx}$ for $\frac{(x+1)^2(x+3)}{\sqrt{x^2+1}}$ at x = 0. (4 marks)

Derivative Applications

Check answers using a graphing calculator.

1. A trough in the cross sectional shape of an inverted equilateral triangle is being filled at a rate of 355 cm^3/min. The trough is 2 m long and has a side length of the triangle of 25 cm. How fast is the water level rising when the water is 12 cm deep? (5 marks)
2. A hotel owner knows that all 400 rooms can be rented for $85 per night. She also knows that for every $5 increase in price, 12 fewer rooms will be rented. How much should she charge per room to maximize her revenue? (5 marks)
3. A 1.85 m tall man is walking toward a 12 m tall street light at night at a rate of 2.2 m/s. How fast is the length of his shadow changing when he is 12 m from the street light? (5 marks)
4. A rectangular prism has its length increasing by 12 cm/min, its width increasing by 4 cm/min and its height increasing by 2 cm/min. How fast is it's volume changing when the dimensions are 200 cm in length, 50 cm in width and 30 cm in height? (5 marks)

Derivatives

1. Determine the derivative of the following functions, and simplify. (3 each)

   a. $f(x) = 2x^2(3x^3 - 4)$
   b. $y = \frac{1}{(5x-1)^3}$
   c. $g(x) = (2x^2 - 3)^{1/3}$

2. Find the derivative of the given functions, and simplify. (4 each)

   a. $f(x) = \frac{(2x-3)^2}{(x^3-7)^3}$

   b. $y = (\sqrt{2x-5})\left(\sqrt[5]{3x^2+4}\right)$

3. Find $\frac{dy}{dx}$ at x = 2 if y = $6u^2$ - 11 and u = $3x^2$ + 2

4. Use implicit differentiation to find $\frac{dy}{dx}$ for xy^2 - yx^2 = 3xy. (3 marks)

5. An object is traveling along a linear path according to the equation s(t) = $4t^3$ - $3t^2$ + 5 where t is measured in seconds and s(t) measured in meters.

   a. How fast is the object moving at t = 4 seconds? (2 marks)

   b. What is the position of the object when it stops moving? (2 marks)

   c. How far has the object traveled when its acceleration is zero? (2 marks)

   d. Is the object moving towards or away from the origin at t = 3 seconds? (3 marks)

Limits

1. Determine the following limits (if they exist). (2 each)

   a. $\lim_{x \to 2} (x^2 + 1)$

   b. $\lim_{x \to -1} \left(\frac{1}{x+1}\right)$

   c. $\lim_{x \to 0} \left(\frac{x^3 - 2x + 7}{3x^2 - 3}\right)$

2. Evaluate the following limits (if they exist). (3 each)

a. $\lim_{x \to 1} \left(\frac{x^2 - 1}{x - 1} \right)$

b. $\lim_{x \to 0} \left(\frac{\sqrt{x + 4} - 2}{x} \right)$

c. $\lim_{x \to 8} \left(\frac{\sqrt[3]{x} - 2}{x - 8} \right)$

3. Find the slope of the following curves at the given value of x. (3 each)

a. $f(x) = 2x^2 - 5$ at $x = -2$

b. $y = x^3 + 3x - 5$ at $x = 1$

Trigonometric Differentiation and Applications

1. Evaluate each of the following limits. Show all reasoning. (2 each)

a. $\lim_{x \to 0} \frac{2\tan^2 x}{x^2}$

b. $\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$

c. $\lim_{x \to 0} \frac{\sin 7x}{\sin 4x}$

d. $\lim_{x \to 0} \frac{\sin(\cos x)}{\sec x}$

2. Determine $\frac{dy}{dx}$. Do not simplify your answer. (3 each)

a. $y = \sec \sqrt[3]{x}$
b. $y = 4\cos^3(\pi x)$
c. $y = 2x(\sqrt{x} - \cot x)$

d. $y = \tan^2(\cos x)$

e. $y = \frac{1}{1+\tan x}$

f. $\sin x + \sin y = 1$

3. Find the maximum and minimum values of f(x) = cox x - sin x, given the interval $-\pi \leq x \leq \pi$. (3 each)

4. A rocket is moving into the air with a height function given by h(t) = 200t2. A camera located 150 m away from the launch site is filming the launch. How fast must the angle of the camera be changing with respect to the horizontal 4 seconds after lift off?
5. The base of an isosceles triangle is 20 cm and the altitude is increasing at the rate of 1 cm/min. At what rate is the base angle increasing when the area is 100 cm^2?