

## CONJECTURES - Discovering Geometry

### Chapter 2

- C-1 Linear Pair Conjecture - If two angles form a linear pair, then the measures of the angles add up to  $180^\circ$ .
- C-2 Vertical Angles Conjecture - If two angles are vertical angles, then they are congruent (have equal measures).
- C-3a Corresponding Angles Conjecture (CA) - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
- C-3b Alternate Interior Angles Conjecture (AIA)- If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
- C-3c Alternate Exterior Angles Conjecture (AEA) - If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.
- C-3 Parallel Lines Conjecture - If two parallel lines are cut by a transversal, then corresponding angles are congruent, alternate interior angles are congruent, and alternate exterior angles are congruent.
- C-4 Converse of the Parallel Lines Conjecture - If two lines are cut by a transversal to form pairs of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are parallel.

### Chapter 3

- C-5 Perpendicular Bisector Conjecture - If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints.
- C-6 Converse of the Perpendicular Bisector Conjecture - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
- C-7 Shortest Distance Conjecture - The shortest distance from a point to a line is measured along the perpendicular segment from the point to the line.
- C-8 Angle Bisector Conjecture - If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
- C-9 Angle Bisector Concurrency Conjecture - The three angle bisectors of a triangle are concurrent (meet at a point).
- C-10 Perpendicular Bisector Concurrency Conjecture - The three perpendicular bisectors of a triangle are concurrent.
- C-11 Altitude Concurrency Conjecture - The three altitudes (or the lines containing the altitudes) of a triangle are concurrent.
- C-12 Circumcenter Conjecture - The circumcenter of a triangle is equidistant from the vertices.
- C-13 Incenter Conjecture - The incenter of a triangle is equidistant from the sides.
- C-14 Median Concurrency Conjecture - The three medians of a triangle are concurrent.
- C-15 Centroid Conjecture - The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is twice the distance from the centroid to the midpoint of the opposite side.
- C-16 Center of Gravity Conjecture - The centroid of a triangle is the center of gravity of the triangular region.

### Chapter 4

- C-17 Triangle Sum Conjecture - The sum of the measures of the angles in every triangle is  $180^\circ$ .
- C-18 Third Angle Conjecture - If two angles of one triangle are equal in measure to two angles of another triangle, then the third angle in each triangle is equal in measure to the third angle in the other triangle.
- C-19 Isosceles Triangle Conjecture - If a triangle is isosceles, then its base angles are congruent.
- C-20 Converse of the Isosceles Triangle Conjecture - If a triangle has two congruent angles, then it is an isosceles triangle.

- C-21 Triangle Inequality Conjecture - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- C-22 Side-Angle Inequality Conjecture - In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
- C-23 Triangle Exterior Angle Conjecture - The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.
- C-24 SSS Congruence Conjecture - If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.
- C-25 SAS Congruence Conjecture - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
- C-26 ASA Congruence Conjecture - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
- C-27 SAA Congruence Conjecture - If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, then the triangles are congruent.
- C-28 Vertex Angle Bisector Conjecture - In an isosceles triangle, the bisector of the vertex angle is also the altitude and the median to the base.
- C-29 Equilateral/Equiangular Triangle Conjecture - Every equilateral triangle is equiangular. Conversely, every equiangular triangle is equilateral.

## Chapter 5

- C-30 Quadrilateral Sum Conjecture - The sum of the measures of the four angles of any quadrilateral is  $360^\circ$ .
- C-31 Pentagon Sum Conjecture - The sum of the measures of the five angles of any pentagon is  $540^\circ$ .
- C-32 Polygon Sum Conjecture - The sum of the measures of the  $n$  interior angles of an  $n$ -gon is  $(n - 2) \cdot 180$ .
- C-33 Exterior Angle Sum Conjecture - For any polygon, the sum of the measures of a set of exterior angles is  $360^\circ$ .
- C-34 Equiangular Polygon Conjecture - You can find the measure of each interior angle of an equiangular  $n$ -gon by using either of these formulas:  $\frac{(n - 2) \cdot 180^\circ}{n}$  or  $\left(180 - \frac{360^\circ}{n}\right)$
- C-35 Kite Angles Conjecture - The non-vertex angles of a kite are congruent.
- C-36 Kite Diagonals Conjecture - The diagonals of a kite are perpendicular.
- C-37 Kite Diagonal Bisector Conjecture - The diagonal connecting the vertex angles of a kite is the perpendicular bisector of the other diagonal.
- C-38 Kite Angle Bisector Conjecture - The vertex angles of a kite are bisected by a diagonal.
- C-39 Trapezoid Consecutive Angles Conjecture - The consecutive angles between the bases of a trapezoid are supplementary.
- C-40 Isosceles Trapezoid Conjecture - The base angles of an isosceles trapezoid are congruent.
- C-41 Isosceles Trapezoid Diagonals Conjecture - The diagonals of an isosceles trapezoid are congruent.
- C-42 Three Midsegments Conjecture - The three midsegments of a triangle divide it into four congruent triangles.
- C-43 Triangle Midsegment Conjecture - A midsegment of a triangle is parallel to the third side and half the length of the third side.
- C-44 Trapezoid Midsegment Conjecture - The midsegment of a trapezoid is parallel to the bases and is equal in length to the average of the lengths of the bases.
- C-45 Parallelogram Opposite Angles Conjecture - The opposite angles of a parallelogram are congruent.
- C-46 Parallelogram Consecutive Angles Conjecture - The consecutive angles of a parallelogram are supplementary.
- C-47 Parallelogram Opposite Sides Conjecture - The opposite sides of a parallelogram are congruent.
- C-48 Parallelogram Diagonals Conjecture - The diagonals of a parallelogram bisect each other.
- C-49 Double-Edged Straightedge Conjecture - If two parallel lines are intersected by a second pair of parallel lines that are the same distance apart as the first pair, then the parallelogram formed is a rhombus.

- C-50 Rhombus Diagonals Conjecture - The diagonals of a rhombus are perpendicular and they bisect each other.
- C-51 Rhombus Angles Conjecture - The diagonals of a rhombus bisect the angles of the rhombus.
- C-52 Rectangle Diagonals Conjecture - The diagonals of a rectangle are congruent and bisect each other.
- C-53 Square Diagonals Conjecture - The diagonals of a square are congruent, perpendicular, and bisect each other.

## Chapter 6

- C-54 Chord Central Angles Conjecture - If two chords in a circle are congruent, then they determine two central angles that are congruent.
- C-55 Chord Arcs Conjecture - If two chords in a circle are congruent, then their intercepted arcs are congruent.
- C-56 Perpendicular to a Chord Conjecture - The perpendicular from the center of a circle to a chord is the bisector of the chord.
- C-57 Chord Distance to Center Conjecture - Two congruent chords in a circle are equidistant from the center of the circle.
- C-58 Perpendicular Bisector of a Chord Conjecture - The perpendicular bisector of a chord passes through the center of the circle.
- C-59 Tangent Conjecture - A tangent to a circle is perpendicular to the radius drawn to the point of tangency.
- C-60 Tangent Segments Conjecture - Tangent segments to a circle from a point outside the circle are congruent.
- C-61 Inscribed Angle Conjecture - The measure of an angle inscribed in a circle is one-half the measure of the central angle.
- C-62 Inscribed Angles Intercepting Arcs Conjecture - Inscribed angles that intercept the same arc are congruent.
- C-63 Angles Inscribed in a Semicircle Conjecture - Angles inscribed in a semicircle are right angles.
- C-64 Cyclic Quadrilateral Conjecture - The opposite angles of a cyclic quadrilateral are supplementary.
- C-65 Parallel Lines Intercepted Arcs Conjecture - Parallel lines intercept congruent arcs on a circle.
- C-66 Circumference Conjecture - If  $C$  is the circumference and  $d$  is the diameter of a circle, then there is a number such that  $C = \pi d$ . If  $d = 2r$  where  $r$  is the radius, then  $C = 2\pi r$ .
- C-67 Arc Length Conjecture - The length of an arc equals the circumference times the measure of the central angle divided by  $360^\circ$ .

## Chapter 7

- C-68 Reflection Line Conjecture - The line of reflection is the perpendicular bisector of every segment joining a point in the original figure with its image.
- C-69 Coordinate Transformations Conjecture
  - The ordered pair rule  $(x, y) \rightarrow (x, -y)$  is a reflection over the  $y$ -axis.
  - The ordered pair rule  $(x, y) \rightarrow (-x, y)$  is a reflection over the  $x$ -axis.
  - The ordered pair rule  $(x, y) \rightarrow (-y, x)$  is a rotation about the origin.
  - The ordered pair rule  $(x, y) \rightarrow (y, x)$  is a reflection over  $y = x$ .
- C-70 Minimal Path Conjecture - If points  $A$  and  $B$  are on one side of line  $\ell$ , then the minimal path from point  $A$  to line  $\ell$  to point  $B$  is found by reflecting point  $B$  over line  $\ell$ , drawing segment  $AB'$ , then drawing segments  $AC$  and  $CB$  where point  $C$  is the point of intersection of segment  $AB'$  and line  $\ell$ .
- C-71 Reflections over Parallel Lines Conjecture - A composition of two reflections over two parallel lines is equivalent to a single translation. In addition, the distance from any point to its second image under the two reflections is twice the distance between the parallel lines.
- C-72 Reflections over Intersecting Lines Conjecture - A composition of two reflections over a pair of intersecting lines is equivalent to a single rotation. The angle of rotation is twice the acute angle between the pair of intersecting reflection lines.
- C-73 Tessellating Triangles Conjecture - Any triangle will create a monohedral tessellation.
- C-74 Tessellating Quadrilaterals Conjecture - Any quadrilateral will create a monohedral tessellation.

## Chapter 8

- C-75 Rectangle Area Conjecture - The area of a rectangle is given by the formula  $A=bh$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the rectangle.
- C-76 Parallelogram Area Conjecture - The area of a parallelogram is given by the formula  $A=bh$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the parallelogram.
- C-77 Triangle Area Conjecture - The area of a triangle is given by the formula  $A = \frac{bh}{2}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the triangle.
- C-78 Trapezoid Area Conjecture - The area of a trapezoid is given by the formula  $A = \frac{(b_1 + b_2)h}{2}$  where  $A$  is the area,  $b_1$  and  $b_2$  are the lengths of the two bases, and  $h$  is the height of the trapezoid.
- C-79 Kite Area Conjecture - The area of a kite is given by the formula  $A = \frac{d_1 \cdot d_2}{2}$  where  $d_1$  and  $d_2$  are the lengths of the diagonals
- C-80 Regular Polygon Area Conjecture - The area of a regular polygon is given by the formula  $A=asn$ , where  $A$  is the area,  $a$  is the apothem,  $s$  is the length of each side, and  $n$  is the number of sides. The length of each side times the number of sides is the perimeter  $P$ , so  $sn = P$ . So the formula for area is also:  
$$A = \frac{a \cdot P}{2}$$
- C-81 Circle Area Conjecture - The area of a circle is given by the formula  $A=\pi r^2$ , where  $A$  is the area and  $r$  is the radius of the circle.
- C-82 The Pythagorean Theorem - In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If  $a$  and  $b$  are the lengths of the legs, and  $c$  is the length of the hypotenuse, then  $a^2+b^2=c^2$ .
- C-83 Converse of the Pythagorean Theorem - If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle.
- C-84 Isosceles Right Triangle Conjecture - In an isosceles right triangle, if the legs have length  $s$ , then the hypotenuse has length  $s \cdot \sqrt{2}$
- C-85 30°-60°-90° Triangle Conjecture - In a 30°-60°-90° triangle, if the shorter leg has length  $a$ , then the longer leg has length  $a \cdot \sqrt{3}$ , and the hypotenuse has length  $2a$ .
- C-86 Distance Formula - The distance between points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is given by  
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
- C-87 Equation of a Circle - The equation of a circle with radius  $r$  and center  $(h, k)$  is  $(x - h)^2 + (y - k)^2 = r^2$

## Chapter 10

- C-88a Conjecture A - If  $B$  is the area of the base of a right rectangular prism and  $H$  is the height of the solid, then the formula for the volume is  $V=BH$ .
- C-88b Conjecture B - If  $B$  is the area of the base of a right prism (or cylinder) and  $H$  is the height of the solid, then the formula for the volume is  $V = BH$ .
- C-88c Conjecture C - The volume of an oblique prism (or cylinder) is the same as the volume of a right prism (or cylinder) that has the same base area and the same height.
- C-88 Prism-Cylinder Volume Conjecture - The volume of a prism or a cylinder is the area of the base multiplied by the height,  $V = B \cdot H$ .
- C-89 Pyramid-Cone Volume Conjecture - If  $B$  is the area of the base of a pyramid or a cone and  $H$  is the height of the solid, then the formula for the volume is  $V = \frac{B \cdot H}{3}$ .
- C-90 Sphere Volume Conjecture - The volume of a sphere with radius  $r$  is given by the formula  $V = \frac{4\pi r^3}{3}$ .

C-91 Sphere Surface Area Conjecture - The surface area, SA, of a sphere with radius r is given by the formula  $SA = 4\pi r^2$

## Chapter 11

- C-92 Dilation Similarity Conjecture - If one polygon is the image of another polygon under a dilation, then the polygons are similar.
- C-93 AA Similarity Conjecture - If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
- C-94 SSS Similarity Conjecture - If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.
- C-95 SAS Similarity Conjecture - If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.
- C-96 Proportional Parts Conjecture - If two triangles are similar, then the corresponding altitudes, medians, and angle bisectors are proportional to the corresponding sides.
- C-97 Angle Bisector/Opposite Side Conjecture - A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the two sides forming the angle.
- C-98 Proportional Areas Conjecture - If corresponding sides of two similar polygons or the radii of two circles compare in the ratio  $\frac{m}{n}$ , then their areas compare in the ratio  $\left(\frac{m}{n}\right)^2$
- C-99 Proportional Volumes Conjecture - If corresponding edges (or radii, or heights) of two similar solids compare in the ratio  $\frac{m}{n}$ , then their volumes compare in the ratio of  $\left(\frac{m}{n}\right)^3$ .
- C-100 Parallel/Proportionality Conjecture - If a line parallel to one side of a triangle passes through the other two sides, then it divides the other two sides proportionally. Conversely, if a line cuts two sides of a triangle proportionally, then it is parallel to the third side.
- C-101 Extended Parallel/Proportionality Conjecture - If two or more lines pass through two sides of a triangle parallel to the third side, then they divide the two sides proportionally.

## Chapter 12

- C-102 SAS Triangle Area Conjecture - The area of a triangle is given by the formula  $A = \left(\frac{1}{2}ab\right)\sin C$ , where a and b are the lengths of two sides and C is the angle between them.
- C-103 Law of Sines - For a triangle with angles A, B, and C and sides of lengths a, b, and c (a is opposite A, b is opposite B, and c is opposite C),  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- C-104 Pythagorean Identity - For any angle A,  $(\sin A)^2 + (\cos A)^2 = 1$
- C-105 Law of Cosines - For any triangle with sides of lengths a, b, and c, and with C the angle opposite the side with length c,  $c^2 = a^2 + b^2 - (2ab)\cos C$