Math 115 Calculus Exam II
29 October 1997

Department of Mathematics
University of Michigan

Name: ______________________ Instructor: ______________________

Signature: ______________________ Section: ______________________

General instructions: Please read the instructions on each individual problem carefully, and indicate answers as directed. Use units wherever appropriate.

This test consists of 10 questions on 13 pages (including this cover sheet and a blank final page), totalling 100 points. When the exam begins, please count all of the pages of the exam, make sure none of them are missing, and write your name on each page.

Problem	Points	Score
1	9	
2	6	
3	12	
4	6	
5	5	
6	6	
7	9	
8	12	
9	15	
10	20	
Total	100	

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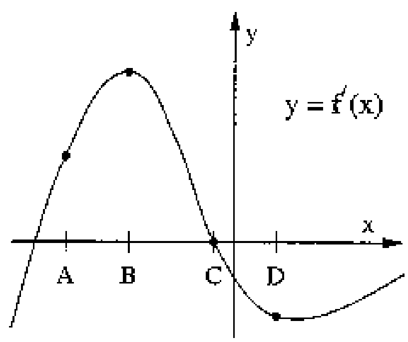
NO PARTIAL CREDIT SECTION. (Problems 1–5.) No explanation necessary; no need to show work.

1. (9 points) The graph below decribes $f'(x)$ **(NOT** $f(x)$**).** At which of the marked values of x is

(a) $f(x)$ greatest? $\boxed{C}$ since f increases for $x < C$ ie $f'(x) > 0$ and decreases for $x > C$ ($f'(x) < 0$)

(b) $f'(x)$ greatest? $\boxed{B}$ from graph

(c) $f''(x)$ greatest? $\boxed{A}$ tangent at A has largest slope.



2. (6 points) Find the equation of the tangent line to the curve $y = 2xe^{x-1}$ at the point $(1, 2)$. Your answer must be **exact**, and not just approximate, for you to receive **full** credit.

Answer: $\boxed{y = 4x - 2}$

$$y = 2xe^{x-1} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(2x)e^{x-1} + 2x\frac{d}{dx}e^{x-1}$$

$$\Rightarrow \frac{dy}{dx} = 2e^{x-1} + 2xe^{x-1}$$

$$x = 1 \Rightarrow y = 2, \quad \frac{dy}{dx} = 4$$

Equation of tangent is

$$y - 2 = 4(x-1) \Rightarrow y = 4x - 2$$

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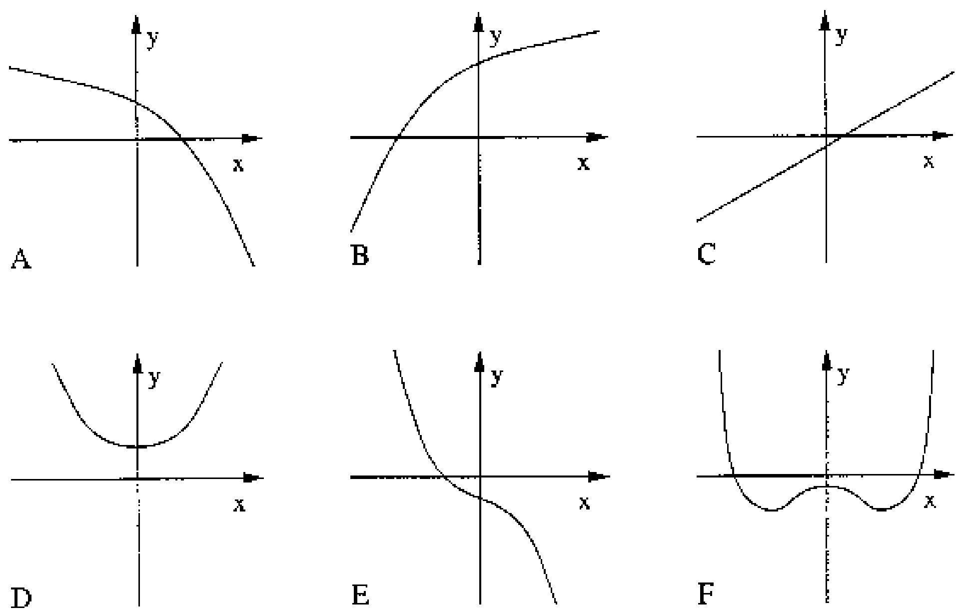
3. (12 points) For each description of a function $f(x)$, write the **single** letter corresponding to the graph of the function described. Note that the horizontal and vertical scales of each graph may be different.

(a) D $\qquad f'(x) < 0$ for $x < 0$; $f'(x) > 0$ for $x > 0$.

(b) B $\qquad f''(0) < 0 < f'(0)$.

(c) F $\qquad f''(0) = -1$, $f'(0) = 0$.



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4. (6 points) Find the **equations** of the horizontal asymptote(s) and vertical asymptote(s) to the graph of the function $y = \dfrac{2x^3}{(x+2)(x-1)}$.

Vertical (of the form $x = a$): $\boxed{X = 1, \ X = -2}$

Horizontal (of the form $y = b$): $\boxed{y = 2}$

vertical given by $y = \infty$ for finite X. $X = -2 \Rightarrow y = \frac{8}{0} = \infty$

$X = 1 \Rightarrow y = \frac{2}{0} = \infty.$

horizontal X large $\Rightarrow y \simeq \frac{2X^2}{X^2} = 2.$

5. (5 points) The Blast-Off Girls are driving to see the Red Hot Sweet Potatoes in Detroit. They travel east for 25 **miles** at an average velocity of 50 miles per hour and then travel another 25 miles east at an average velocity of 75 miles per hour. What is their average velocity for the whole trip? Round off your answer to the nearest tenth of a mile per hour, and make sure you use the correct **units**.

Answer: $\boxed{60 \text{ mph.}}$

Time taken for trip $= \frac{25}{50} + \frac{25}{75}$ hours

$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ hours.

Distance $= 25 + 25 = 50$ miles.

Average velocity $= \frac{\text{Distance}}{\text{Time}} = \frac{50}{5/6} = 60$ mph.

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6. (6 points) At what point P on the curve $y = x\sqrt{x}$ does the tangent line to the curve have slope 3? Round off your answer to 2 decimal places.

$P = (\boxed{4}, \boxed{8})$

$$y = x^{3/2} \Rightarrow \frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\frac{dy}{dx} = 3 \Rightarrow \frac{3}{2} x^{1/2} = 3 \Rightarrow x^{1/2} = 2$$

$$\Rightarrow x = 4$$

$$\Rightarrow y = 4\sqrt{4} = 8.$$

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SHORT ANSWER SECTION. (Problems 7–9.) Limited partial credit may be possible, and a few sentences of explanation may be required. In addition, you may be asked to state your final answer in the form of a complete sentence.

7. The musical group Mr. Largebeat is playing a free concert at the West Park Bandshell. Let $f(d)$ be the number of people who will attend the concert if Mr. Largebeat is playing at a sound level of d decibels.

(a) (5 points) **EXPLAIN** what $f'(85) = 10$ means in practical terms, using no more than **TWO** sentences. The numbers 85 and 10 should appear in your answer.

At 85 decibels the number of people attending the concert is increasing at a rate of 10 people per decibel.

(b) (4 points) Suppose $f'(105) = -8$. Based on this information, if Mr. Largebeat is currently playing at 105 decibels, and they want to increase their attendance, should they play slightly louder or slightly softer? **EXPLAIN** your answer in no more than **TWO** sentences.

At 105 decibels the number of people attending the concert decreases at a rate of 8 people per decibel increase (since $f'(105) = -8 < 0$).
Hence to increase the attendance at the concert they should play softer.

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8. Let $k(x)$ be a function described by the following table.

x	-4.02	-4.01	-4.00	-3.99	-3.98
$k(x)$	5.4076	5.3965	5.3877	5.3789	5.3622

(a) (5 points) Estimate $k'(-4)$ as accurately as possible using the given data. **NO EXPLANATION NECESSARY**, but show all your work, and round off your final answer to 2 decimal places.

$$k'(-4) \simeq \frac{k'(-4+h) - k'(-4)}{h}$$

$$h = .01 \Rightarrow k'(-4) \simeq \frac{k'(-3.99) - k'(-4)}{.01}$$

$$= \frac{5.3789 - 5.3877}{.01} = -.88$$

$$h = -.01 \Rightarrow k'(4) \simeq \frac{k'(-4.01) - k'(-4)}{-.01} = -.88$$

(b) (7 points) Let $g(x) = -2\sin(k(x))$. Estimate $g'(-4)$ as accurately as possible using the given data. **NO EXPLANATION NECESSARY**, but show all your work, and round off your final answer to 2 decimal places.

$$g'(-4) \simeq \frac{g(-4+h) - g(-4)}{h}$$

$$= \frac{-2\sin(k(-4+h)) + 2\sin(k(-4))}{h}$$

$$h = .01 \Rightarrow g'(4) \simeq \frac{-2\sin(k(-3.99)) + 2\sin(k(-4))}{.01}$$

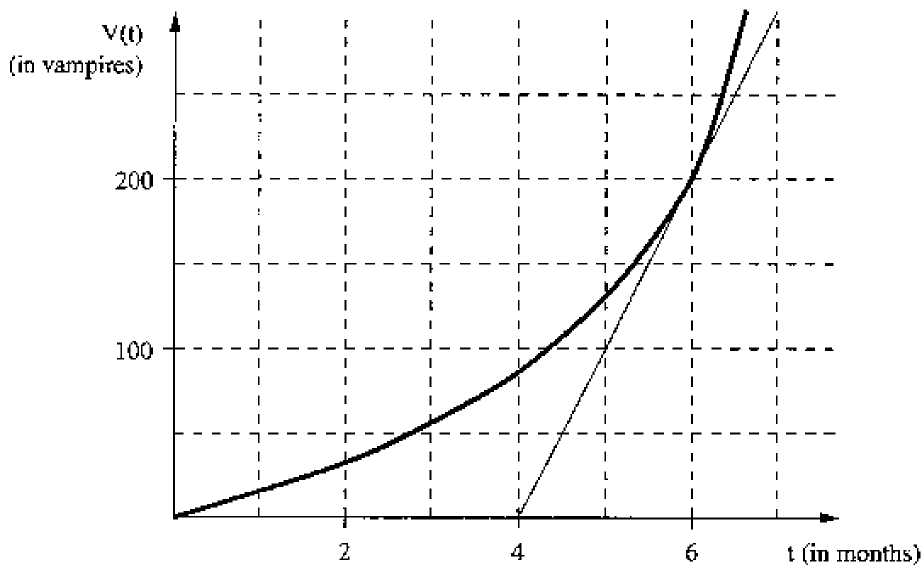
$$= \frac{-2\sin(5.3789) + 2\sin(5.3877)}{.01}$$

$$= 1.094$$

Answer: $g'(-4) \simeq 1.09$

9. This question has parts (a) and (b), on two pages.

Muffy the Vampire-Hunting Calculus Student is making a graph of $V(t)$, the total number of vampires she has slain since she moved to Ann Arbor, as a function of t, the number of months that she has lived in Ann Arbor. The result is the dark solid curve shown below; the lighter solid line is the tangent line to the curve at (6, 200).



(a) (6 points) Calculate $V'(6)$, showing enough work for us to tell how you did your calculation; and **EXPLAIN** the practical meaning of the value of $V'(6)$, using no more than **TWO** complete sentences. Make sure you use the correct **units** in your answer, and round off your final numerical answer to the nearest integer.

$V'(6) =$ slope of line $= \dfrac{200 - 0}{6 - 4}$

$= \dfrac{200}{2} = 100.$

Practical meaning is: after 6 months Muffy is killing vampires at a rate of 100 per month.

(Problem continued on next page.)

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(b) (9 points) Let $A(t) = \frac{V(t)}{t}$ be the **average** number of vampires Muffy has slain per month between the time she moved to Ann Arbor and t months later. Calculate the rate of change of $A(t)$ at $t = 6$. **NO EXPLANATION** necessary, but show all your work. State your final answer in the form of a complete sentence, using the correct **units**, and rounding off your final numerical answer to the nearest integer.

$$A'(t) = \frac{1}{t}\frac{d}{dt}V(t) + V(t)\frac{d}{dt}\left(\frac{1}{t}\right)$$

$$= \frac{V'(t)}{t} - \frac{V(t)}{t^2}.$$

$$A'(6) = \frac{V'(6)}{6} - \frac{V(6)}{36}$$

$$= \frac{100}{6} - \frac{200}{36} = 100\left[\frac{1}{6} - \frac{1}{18}\right]$$

$$= \frac{100}{9} = 11\ 1/9.$$

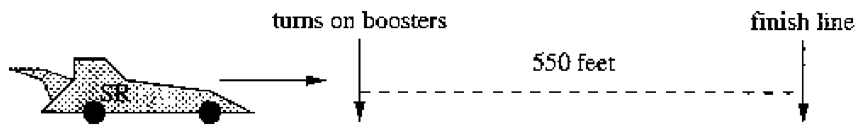
After 6 months the average number of vampires per month killed by Muffy is increasing at a rate of 11 per month.

ESSAY QUESTION

EXPLAIN your answers using complete sentences. Use graphs (labeled carefully and neatly), tables, and/or formulas in your explanations if possible.

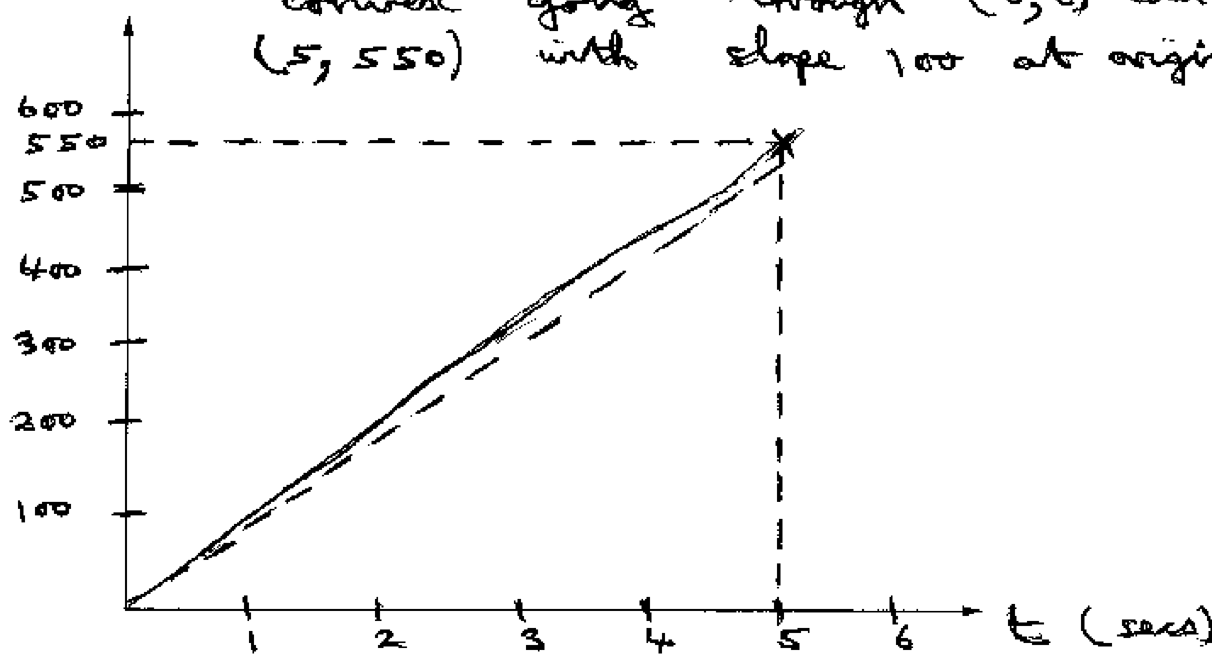
10. This question has parts (a), (b), and (c), on three pages.

Speedy Racer and her arch-rival Racer Yam enter the home stretch of the Ann Arbor 500 race completely even. When Speedy is 550 feet from the finish line, traveling at a velocity of 100 feet per second, she activates her secret boosters. Her velocity increases and continues to increase until she crosses the finish line 5 seconds later, winning the race.



(a) (6 points) Let $s(t)$ be the number of feet that Speedy has traveled between the time when she activates her secret boosters and t seconds later. On the axes below, draw a graph of $s(t)$ for $0 \le t \le 5$, making sure to display all of the information given above. Remember to label your axes neatly, clearly, and with the correct **units**.

Graph should be slightly convex going through (0,0) and (5,550) with slope 100 at origin.



(Problem continued on next page.)

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(b) (8 points) Using only the information above, one can conclude that at $t = 3$ seconds after activating her secret boosters, Speedy Racer has traveled

at least $\boxed{300}$ feet and at most $\boxed{330}$ feet.

Fill in the blanks in with the best answers which can be obtained by using **only** the given information, and **EXPLAIN** your answers.

velocity is increasing hence always larger than initial velocity of 100 ft/sec. Hence in 3 secs go at least 300 ft.

velocity is increasing hence if x = distance gone in first 3 secs then distance gone in following 2 secs is at least $\frac{2}{3}x$. Hence

$$x + \frac{2}{3}x \leq 550 \Rightarrow$$

$$x \leq \frac{3}{5} * 550 = 330.$$

(Problem continued on next page.)

(c) (6 points) When Speedy crosses the finish line, could she be traveling at a velocity of 105 feet per second? **EXPLAIN** your answer.

If she is travelling 105 ft/sec across the finish line the total distance she has gone is less than 5 * 105 = 525 ft. Since we know she travels 550 ft. her final speed must be greater than 105.