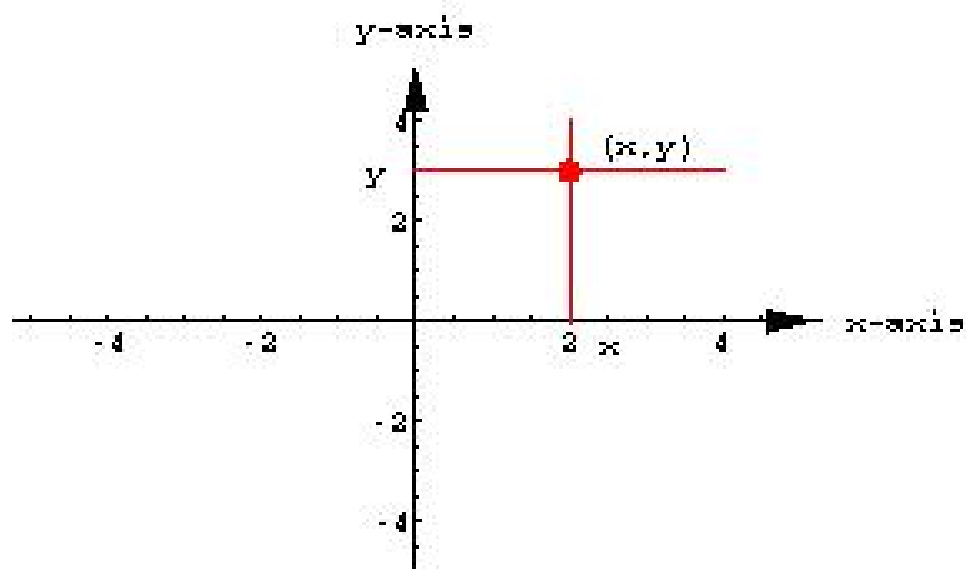


1.2. GRAPHS OF RELATIONS

Graphs of relations as sets in coordinate plane

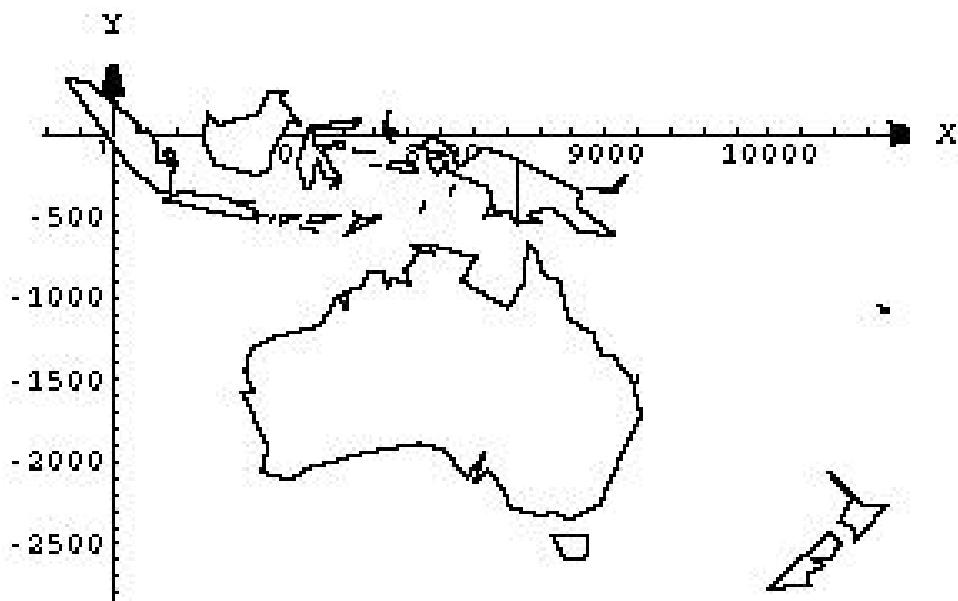
Let us recall that a coordinate plane is formed by choosing two number lines (lines where points represent real numbers) which intersect at the right angle. Usually, one line is horizontal and the other vertical. The horizontal line is the x -axis and vertical line y -axis. The point of intersection of the two lines is called the origin. In a coordinate plane each point represents uniquely an ordered pair of real numbers (x, y) and each ordered pair of real numbers is represented by a point. If a point P represents the pair (x, y) then x is called x -coordinate and y is called y -coordinate of the point P .



1.2.1. DEFINITION.

The graph of a relation R is the set of all points (x, y) in a coordinate plane such that x is related to y through the relation R .

1.2.2. EXAMPLE. The map of Australia below graphically represents a relation since any set in a coordinate plane does it.

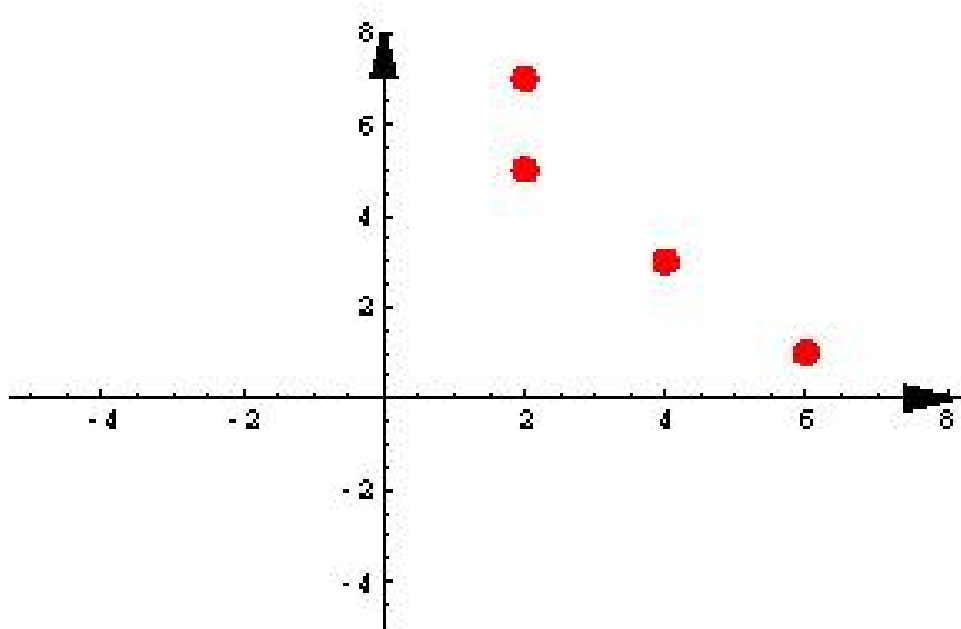


Graphing relations

A relation consisting of finitely many ordered pairs of numbers could be graphed by simple plotting of points.

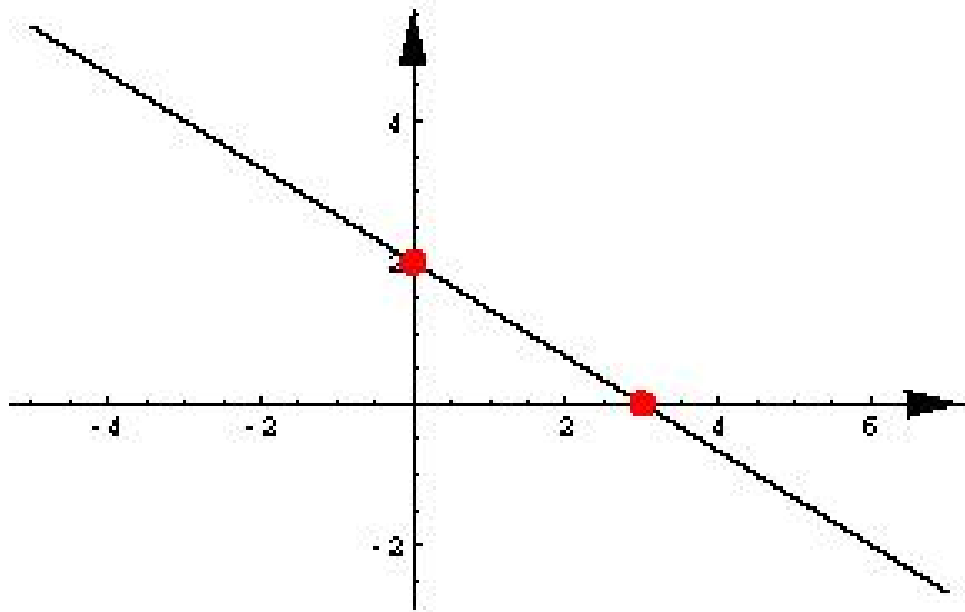
1.2.3. EXAMPLE. The figure below shows the graph of the relation

$$R = \{(2, 5), (4, 3), (6, 1), (2, 7)\}.$$

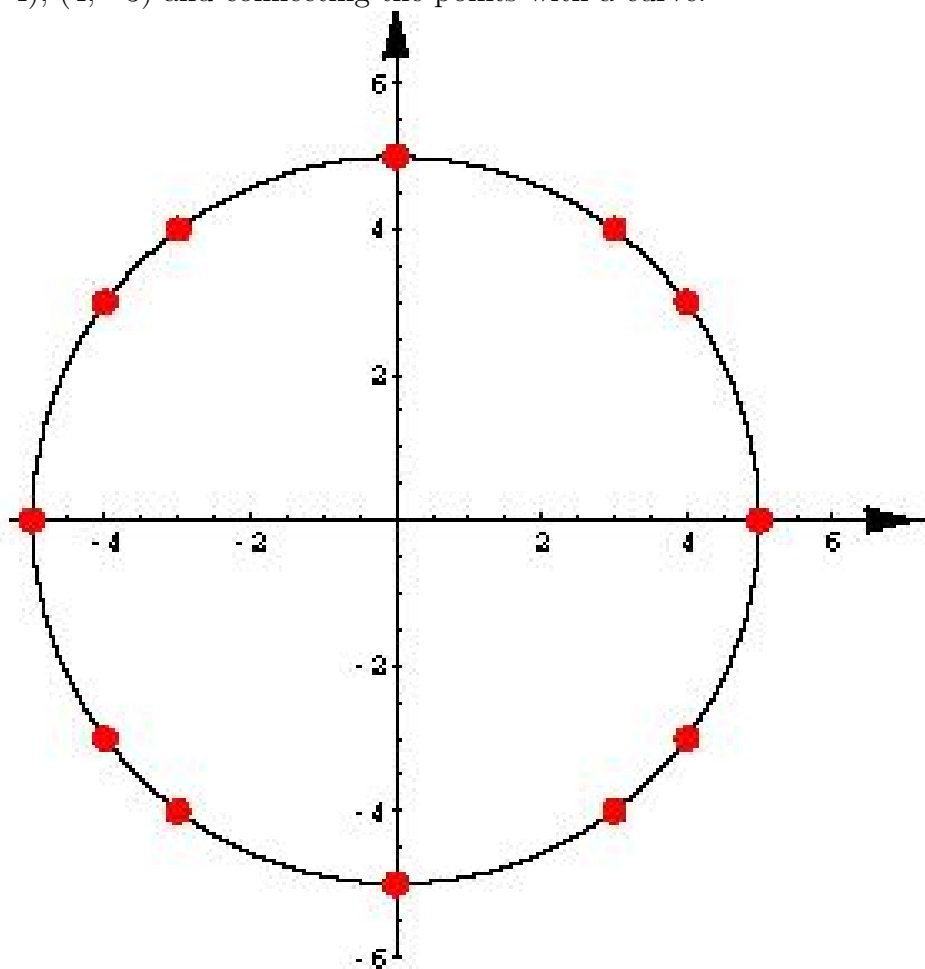


For a relation given by an equation we plot enough points to reveal the essential behavior of the graph.

1.2.4. EXAMPLE. The figure below shows the graph of the relation $2x + 3y = 6$. The graph could be sketched by plotting the points $(0, 2)$ and $(3, 0)$ and connecting the points with a straight line.



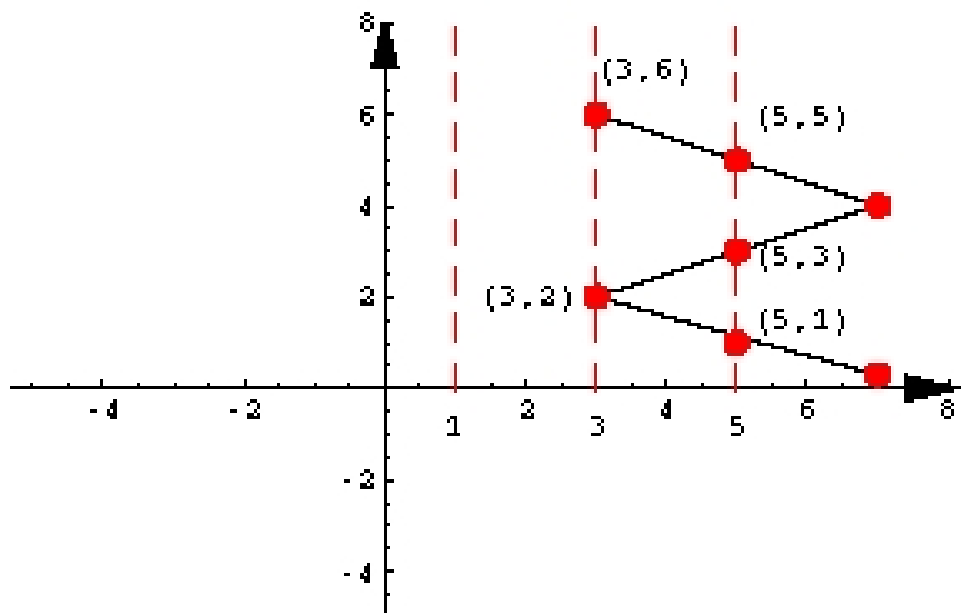
1.2.5. EXAMPLE. The figure below shows the graph of the relation $x^2 + y^2 = 25$. The graph could be sketched by plotting the points $(5, 0)$, $(4, 3)$, $(3, 4)$, $(0, 5)$, $(-3, 4)$, $(-4, 3)$, $(-5, 0)$, $(-4, -3)$, $(-3, -4)$, $(0, -5)$, $(3, -4)$, $(4, -3)$ and connecting the points with a curve.



Identifying relations given graphically

Any graph in a coordinate plane defines a relation in the following way: if a point representing the ordered pair (x, y) lies on the graph then x is related to y and if the point representing the pair (x, y) does not belong to the graph then x is not related to y . The next example illustrates the above definition.

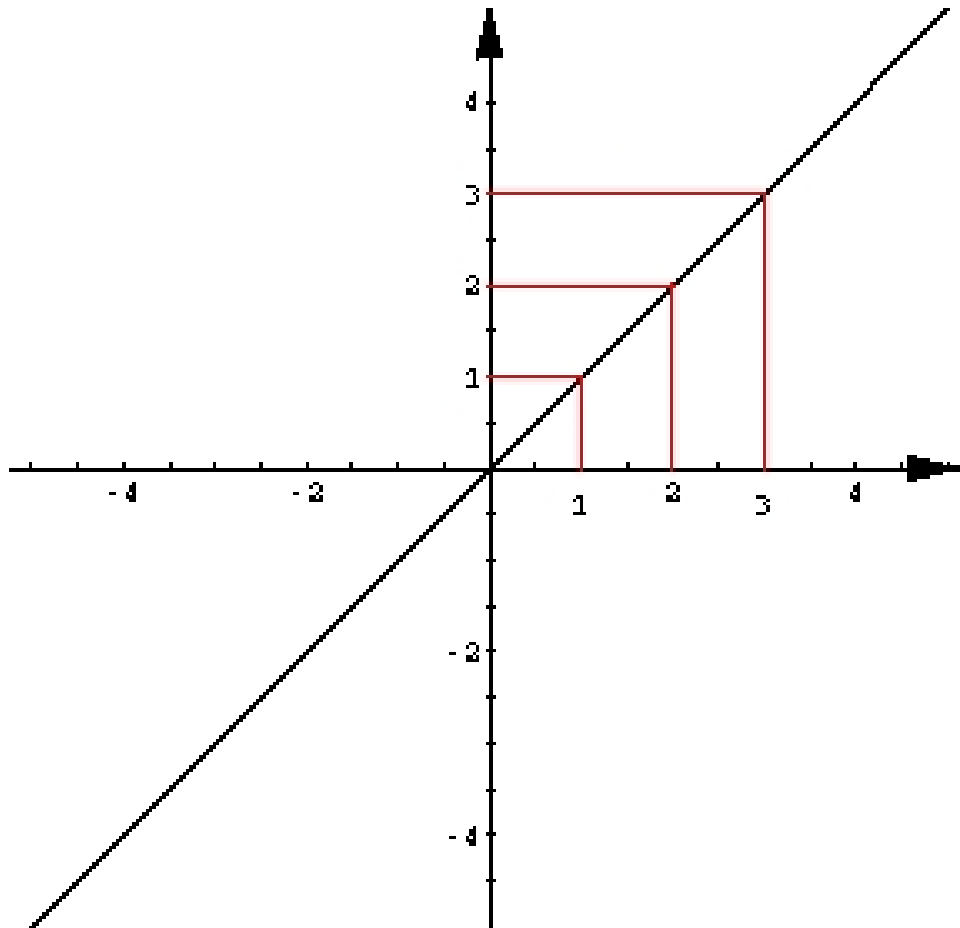
1.2.6. EXAMPLE. In the figure below the graph of the letter Z in the coordinate plane represents a relation.



The graph shows that the number 5 is related to the numbers 1, 3, 5 because the vertical line passing through the point representing 5 in the x-axis intersects the graph at the points which have the y-coordinates 1, 3, 5, respectively. In the same way we see that 3 is related to 2 and 6. The number 1 is not related to any number because the vertical line passing through the point 1 on the x-axis does not intersect the graph.

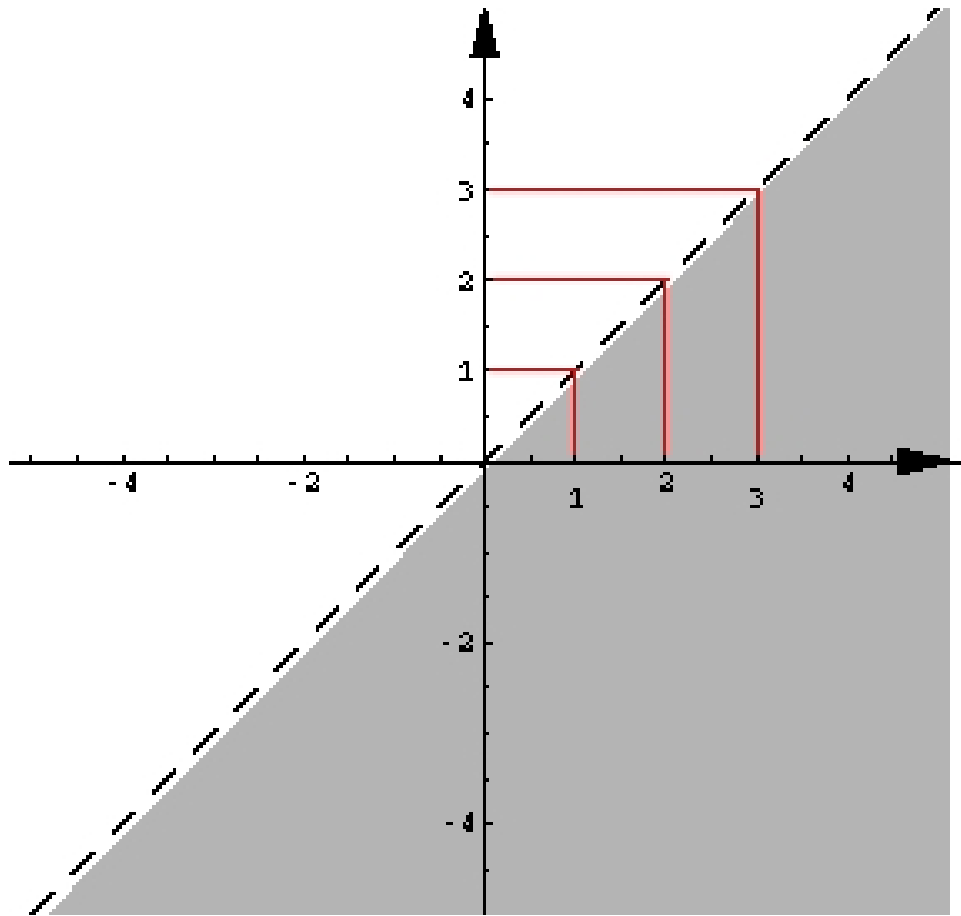
1.2.7. EXERCISES I.

1. Exercise. Describe the relation represented by the graph below.



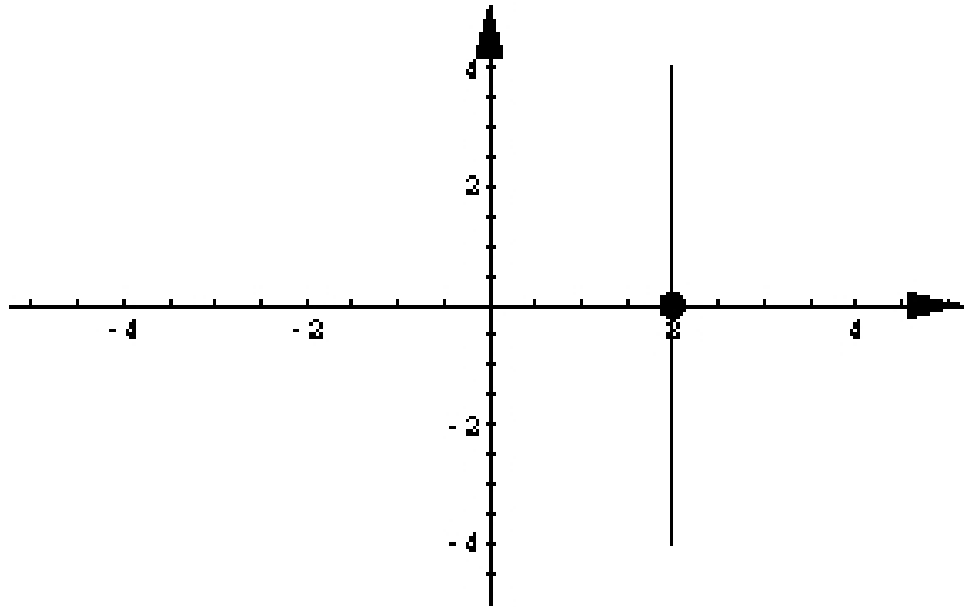
Go to answer 1

2. Exercise. Describe the relation represented by the graph below.



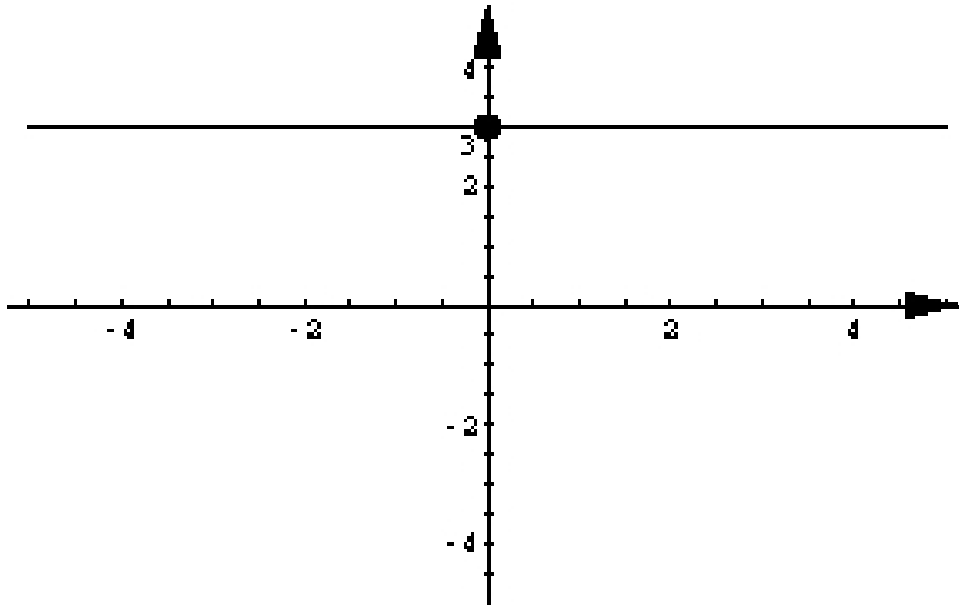
Go to answer 2

- Exercise. Describe the relation represented by the graph below.



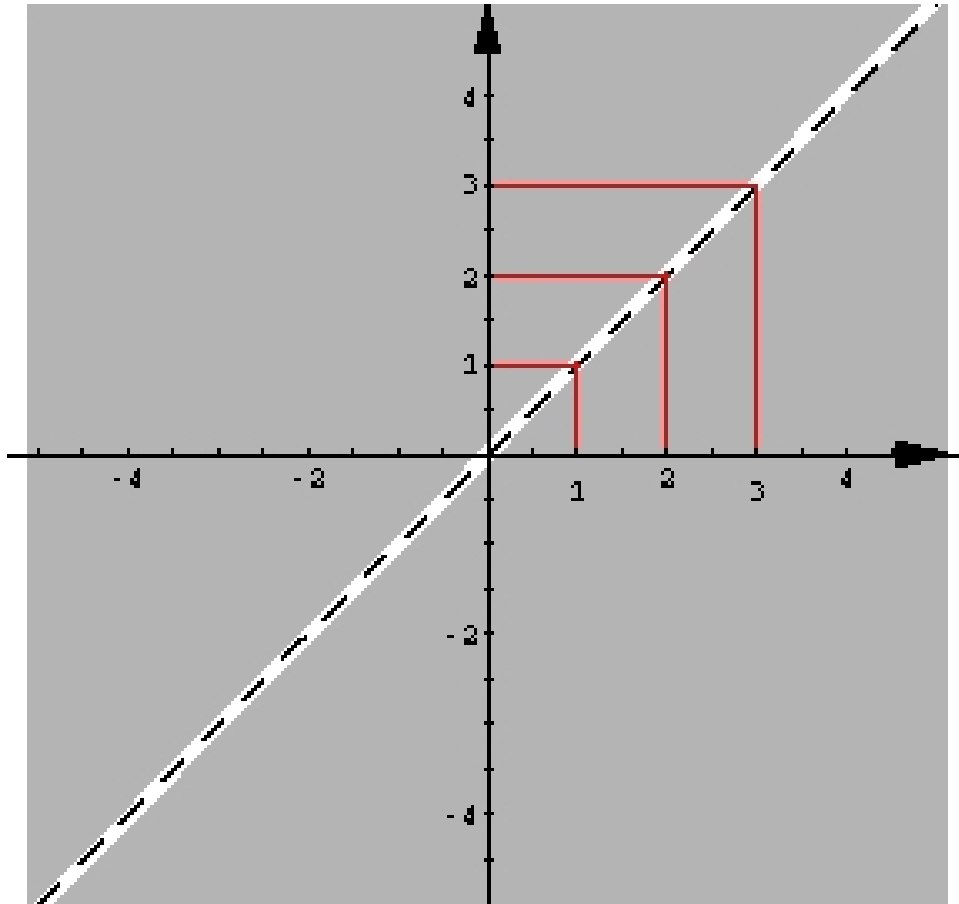
Go to answer 3

4. Exercise. Describe the relation represented by the graph below.



Go to answer 4

5. Exercise. Describe the relation represented by the shaded region shown below.



Go to answer 5

1.2.8. ANSWERS I.

1. Answer to Exercise 1. The graph represents the relation ' $x = y$ '.

Go back 1

2. Answer to Exercise 2. The graph represents the relation ' $x > y$ '.

Go back 2

3. Answer to Exercise 3. Since all points on the graph have the x-coordinate equal to 2 the graph represents the relation '2 is related to each number y '.

Go back 3

4. Answer to Exercise 4. Since all points on the graph have the y-coordinate equal to 3 the graph represents the relation 'each number x is related to the number 3'.

Go back 4

5. Answer to Exercise 5. The shade region consists of all points but the points on the line $x = y$. So the graph describes the relation ' x is related to y if $x \neq y$ '.

Go back 5

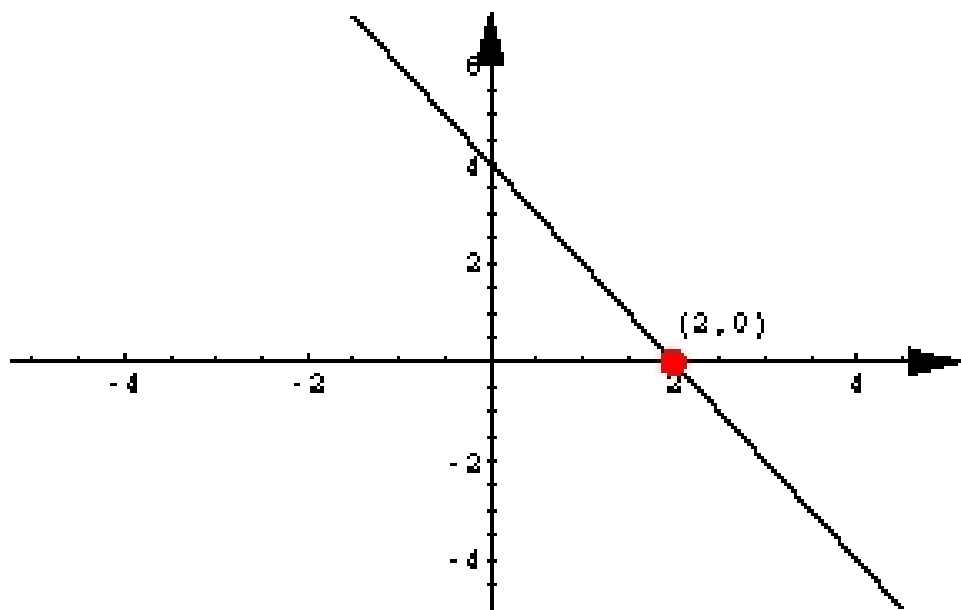
Finding intercepts of a graph

Obviously, when we sketch a graph in a coordinate plane, the points of intersection of the graph with the axes are essential.

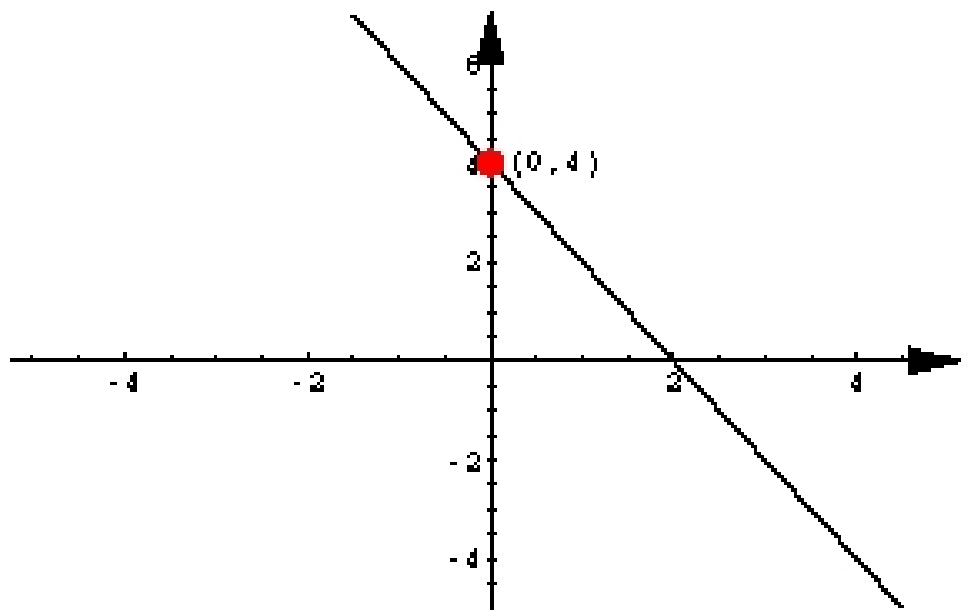
1.2.9. DEFINITION.

The x-intercepts of a graph are the points at which the graph intersects the x-axis and the y-intercepts of a graph are the points at which the graph intersects the y-axis.

1.2.10. EXAMPLE. To find the x-intercepts, let y be zero and solve for x . For the relation given by the equation $2x + y = 4$ substitution $y = 0$ gives us $x = 2$ so the x-intercept is $(2, 0)$.

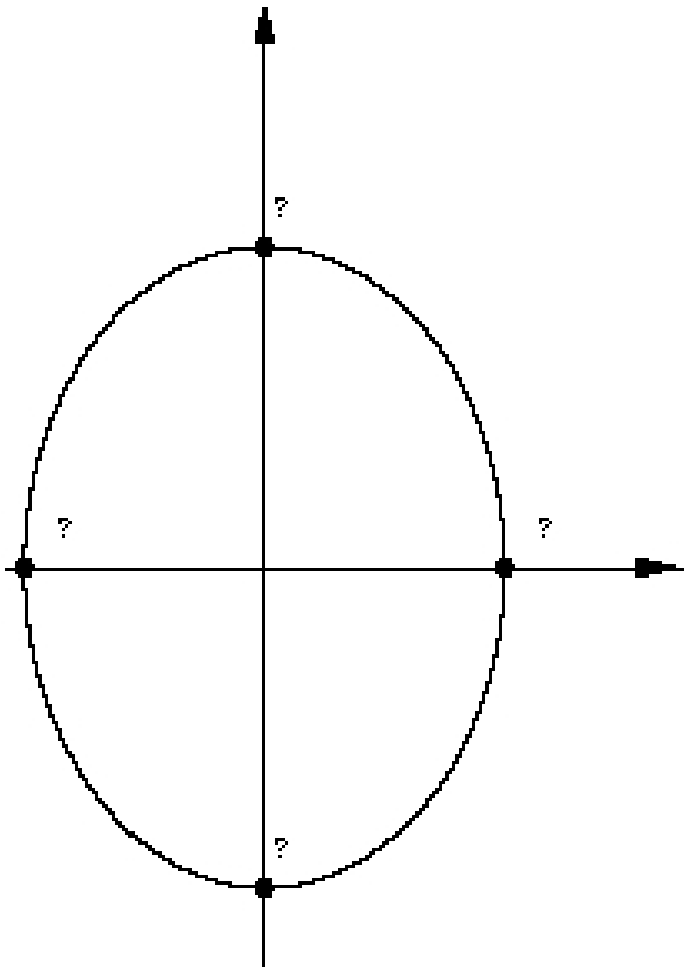


1.2.11. EXAMPLE. To find the y -intercepts, let x be zero and solve for y . For the relation given by the equation $2x + y = 4$ substitution $x = 0$ gives us $y = 4$ so the y -intercept is $(0, 4)$.



1.2.12. EXERCISES II.

1. Exercise. Find the intercepts of the graph of the equation $16x^2 + 9y^2 = 144$.



Go to answer 1

1.2.13. ANSWERS II.

1. Answer to Exercise 1.

The x-intercepts are $(3, 0)$ and $(-3, 0)$. The y-intercepts are $(4, 0)$ and $(-4, 0)$. Substituting $y = 0$ we obtain $16x^2 + 9(0)^2 = 144$. It means that $x^2 = 9$ and $x = 3$ or $x = -3$. The y-intercepts $(0, 4)$ and $(0, -4)$ are obtained in the same way by substituting $x = 0$ and solving the equation $16(0)^2 + 9y^2 = 144$.

Go back 1