# **1.3. DOMAIN AND RANGE**

#### Defining domain and range of relation

A relation R between the elements of a set X and the elements of a set Y is the set of pairs (x, y) where x is an element of X and y is an element of Y. The relations nay not include all pairs giving us a correspondence between some values of x and some values of y only. There are always two sets associated with a relation R:

(1) the set of values of the variable x which have a pair in the relation R;

(2) the set of values of the variable y which have a pair in the relation R. Below we give more precise definition.

#### 1.3.1. DEFINITION.

Let R be a relation. Then R is a subset of the set of all pairs

 $\{(x,y)|x \text{ belongs } X \text{ and } y \text{ belongs to } Y\}.$ 

The domain of R is the set

 $\{x | x \text{ belongs to } X \text{ and there exists } y \text{ in } Y \text{ such that } x \text{ is related to } y\}.$ 

The range of R is the set

 $\{y|y \text{ belongs to } Y \text{ and there exists } x \text{ in } X \text{ which is related to } y\}.$ 

1.3.2. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the domain of the relation because the vertical line passing through 5 in the x-axis intersects the region. The same is true for each number between 1 and 7 including 1 and 7. So the domain is the closed interval [1, 7].

1.3.3. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the range of the relation because the horizontal line passing through 5 in the y-axis intersects the region. The same is true for each number between 2 and 6 including 2 and 6. So the range is the closed interval [2, 6].

# Finding domains and ranges of relations

# 1.3.4. EXERCISES.

1. Exercise. Find the domain and the range of the relation

$$R = \{(2,5), (4,3), (6,1), (2,7)\}.$$

Go to answer 1

- 2. Exercise. Find the domain and the range of the relation by the equation 2x + 3y = 5. Go to answer 2
- 3. Exercise. Find the domain and the range of the relation by the equation xy = 1.

Go to answer 3

4. Exercise. Find the domain and the range of the relation by the equation  $y = x^2 - 3$ .

Go to answer 4

5. Exercise. Find the domain and the range of the relation by the equation  $y=\frac{x}{x-2}$  .

Go to answer 5

6. Exercise. Find the domain and the range of the relation by the equation  $y^2 = x - 3$ .

Go to answer 6

#### 1.3.7. ANSWERS.

1. Answer to Exercise 1. The domain of R is 2, 4, 6 because the numbers 2, 4, 6 appear as the first elements of the pairs in R. The range of R is  $\{5,3,1,7\}$  because the numbers 5, 3, 1, 7 appear as the second elements of the pairs in R.

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2. Answer to Exercise 2. The domain of R is the set of all real numbers. If x is a real number then solving the equation for y we see that x is related to  $y = \frac{5}{3} - \frac{2x}{3}$ . For instance x = 2 is related to  $y = \frac{1}{3}$ . The range of R is the set of all real numbers. If y is a real number then solving the equation for x we obtain that  $x = \frac{5}{2} - \frac{3y}{2}$  is related to y. For instance if y = 3 then x = -2 is related to y = 3.

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- 3. Answer to Exercise 3. The domain and the range of R is the set of all real numbers except for the number 0. We explain how to find the domain only. If x = 0 then for every value of y we have 0y = 0. It means that there is no value of y such that 0y = 1. Thus the number 0 does not belong to the domain. If  $x \neq 0$  then x is related to  $y = \frac{1}{x}$ . Go back 3
- 4. Answer to Exercise 4. The domain of R is the set of all real numbers because for every value of x the number x is related to  $y = x^2 3$ . The range of R is the interval [-3, x). If  $x^2 \ge 0$  then  $x^2 3 \ge -3$  and  $y \ge -3$ . So we see that if y < -3 then there is no x such that  $y = x^2 3$ . It means that y does not belong to the range. If  $y \ge -3$  then  $y + 3 \ge 0$  and the square root of y + 3 is defined. So x equal to  $\sqrt{y + 3}$  is related to y. Go back 4
- 5. Answer to Exercise 5. The domain is the set of all real numbers but, the number 2 because substitution x = 2 leads to dividing by 0. The range is the set of all real numbers but the number 1 because after solving the equation for x we obtain  $x = \frac{2y}{(y-1)}$  which is undefined for y = 1.

Go back 5

6. Answer to Exercise 6. The domain is the interval [3, X) and the range is the set of all real numbers. Since for every value of y we have  $y^2 \ge 0$  the value of x needs to satisfy the inequality  $x - 3 \ge 0$  which gives  $x \ge 3$ .

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