1.3. DOMAIN AND RANGE

Defining domain and range of relation

A relation R between the elements of a set X and the elements of a set Y is the set of pairs (x, y) where x is an element of X and y is an element of Y. The relations nay not include all pairs giving us a correspondence between some values of x and some values of y only. There are always two sets associated with a relation R:

(1) the set of values of the variable x which have a pair in the relation R;

(2) the set of values of the variable y which have a pair in the relation R. Below we give more precise definition.

1.3.1. DEFINITION.

Let R be a relation. Then R is a subset of the set of all pairs

$$\{(x, y)|x \textit{ belongs } X \textit{ and } y \textit{ belongs to } Y\}.$$

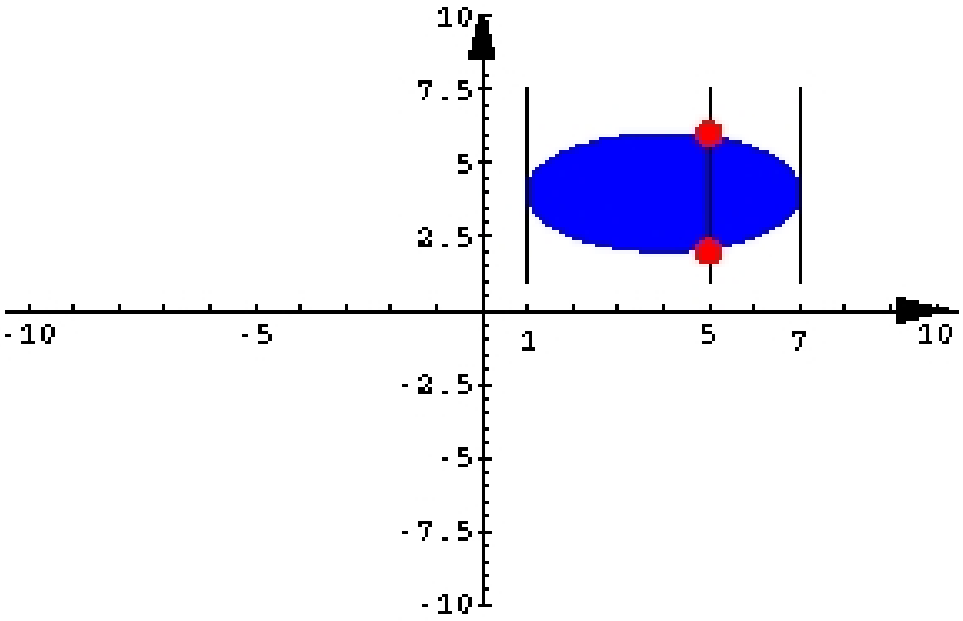
The domain of R is the set

$$\{x|x \textit{ belongs to } X \textit{ and there exists } y \textit{ in } Y \textit{ such that } x \textit{ is related to } y\}.$$

The range of R is the set

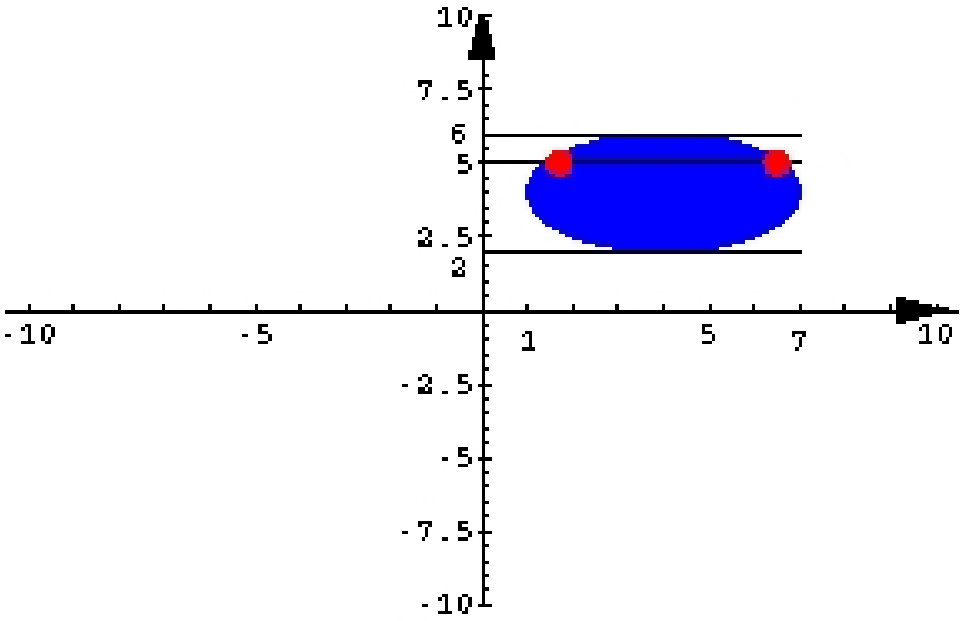
$$\{y|y \textit{ belongs to } Y \textit{ and there exists } x \textit{ in } X \textit{ which is related to } y\}.$$

1.3.2. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the domain of the relation because the vertical line passing through 5 in the x-axis intersects the region. The same is true for each number between 1 and 7 including 1 and 7. So the domain is the closed interval $[1, 7]$.

1.3.3. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the range of the relation because the horizontal line passing through 5 in the y-axis intersects the region. The same is true for each number between 2 and 6 including 2 and 6. So the range is the closed interval $[2, 6]$.

1.3.4. EXERCISES.

1. Exercise. Find the domain and the range of the relation

$$R = \{(2,5),(4,3),(6,1),(2,7)\}.$$

Go to answer 1

2. Exercise. Find the domain and the range of the relation by the equation $2x + 3y = 5$.

Go to answer 2

3. Exercise. Find the domain and the range of the relation by the equation $xy = 1$.

Go to answer 3

4. Exercise. Find the domain and the range of the relation by the equation $y = x^2 - 3$.

Go to answer 4

5. Exercise. Find the domain and the range of the relation by the equation $y = \frac{x}{x-2}$.

Go to answer 5

6. Exercise. Find the domain and the range of the relation by the equation $y^2 = x - 3$.

Go to answer 6

1.3.7. ANSWERS.

1. Answer to Exercise 1. The domain of R is $2, 4, 6$ because the numbers 2, 4, 6 appear as the first elements of the pairs in R. The range of R is $\{5, 3, 1, 7\}$ because the numbers 5, 3, 1, 7 appear as the second elements of the pairs in R.

   Go back 1

2. Answer to Exercise 2. The domain of R is the set of all real numbers. If x is a real number then solving the equation for y we see that x is related to $y = \frac{5}{3} - \frac{2x}{3}$. For instance $x = 2$ is related to $y = \frac{1}{3}$. The range of R is the set of all real numbers. If y is a real number then solving the equation for x we obtain that $x = \frac{5}{2} - \frac{3y}{2}$ is related to y. For instance if $y = 3$ then $x = -2$ is related to $y = 3$.

   Go back 2

3. Answer to Exercise 3. The domain and the range of R is the set of all real numbers except for the number 0. We explain how to find the domain only. If $x = 0$ then for every value of y we have $0y = 0$. It means that there is no value of y such that$0y = 1$. Thus the number 0 does not belong to the domain. If $x \neq 0$ then x is related to $y = \frac{1}{x}$.

   Go back 3

4. Answer to Exercise 4. The domain of R is the set of all real numbers because for every value of x the number x is related to $y = x^2 - 3$. The range of R is the interval $[-3, x)$. If $x^2 \geq 0$ then $x^2 - 3 \geq -3$ and $y \geq -3$. So we see that if $y < -3$ then there is no x such that $y = x^2 - 3$. It means that y does not belong to the range. If $y \geq -3$ then $y + 3 \geq 0$ and the square root of $y + 3$ is defined. So x equal to $\sqrt{y+3}$ is related to y.

   Go back 4

5. Answer to Exercise 5. The domain is the set of all real numbers but, the number 2 because substitution $x = 2$ leads to dividing by 0. The range is the set of all real numbers but the number 1 because after solving the equation for x we obtain $x = \frac{2y}{(y-1)}$ which is undefined for $y = 1$.

   Go back 5

6. Answer to Exercise 6. The domain is the interval $[3, X)$and the range is the set of all real numbers. Since for every value of y we have $y^2 \geq 0$ the value of x needs to satisfy the inequality $x - 3 \geq 0$ which gives $x \geq 3$.

   Go back 6