### 1.3. DOMAIN AND RANGE

Defining domain and range of relation
A relation $R$ between the elements of a set $X$ and the elements of a set $Y$ is the set of pairs $(x, y)$ where $x$ is an element of $X$ and $y$ is an element of $Y$. The relations nay not include all pairs giving us a correspondence between some values of $x$ and some values of $y$ only. There are always two sets associated with a relation $R$ :
(1) the set of values of the variable $x$ which have a pair in the relation $R$;
(2) the set of values of the variable $y$ which have a pair in the relation $R$. Below we give more precise definition.

### 1.3.1. DEFINITION.

Let $R$ be a relation. Then $R$ is a subset of the set of all pairs

$$
\{(x, y) \mid x \text { belongs } X \text { and } y \text { belongs to } Y\} \text {. }
$$

The domain of $R$ is the set
$\{x \mid x$ belongs to $X$ and there exists $y$ in $Y$ such that $x$ is related to $y\}$.
The range of $R$ is the set
$\{y \mid y$ belongs to $Y$ and there exists $x$ in $X$ which is related to $y\}$.

### 1.3.2. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the domain of the relation because the vertical line passing through 5 in the x -axis intersects the region. The same is true for each number between 1 and 7 including 1 and 7 . So the domain is the closed interval $[1,7]$.

### 1.3.3. EXAMPLE.



In the above figure the oval-shaped region represents a relation and we can see that the number 5 belongs to the range of the relation because the horizontal line passing through 5 in the y -axis intersects the region. The same is true for each number between 2 and 6 including 2 and 6 . So the range is the closed interval $[2,6]$.

## Finding domains and ranges of relations

### 1.3.4. EXERCISES.

1. Exercise. Find the domain and the range of the relation

$$
R=\{(2,5),(4,3),(6,1),(2,7)\}
$$

Go to answer 1
2. Exercise. Find the domain and the range of the relation by the equation $2 x+3 y=5$.

Go to answer 2
3. Exercise. Find the domain and the range of the relation by the equation $x y=1$.

Go to answer 3
4. Exercise. Find the domain and the range of the relation by the equation $y=x^{2}-3$.

Go to answer 4
5. Exercise. Find the domain and the range of the relation by the equation $y=\frac{x}{x-2}$.

Go to answer 5
6. Exercise. Find the domain and the range of the relation by the equation $y^{2}=x-3$.

Go to answer 6

### 1.3.7. ANSWERS.

1. Answer to Exercise 1. The domain of $R$ is $2,4,6$ because the numbers $2,4,6$ appear as the first elements of the pairs in $R$. The range of $R$ is $\{5,3,1,7\}$ because the numbers $5,3,1,7$ appear as the second elements of the pairs in $R$.
Go back 1
2. Answer to Exercise 2. The domain of $R$ is the set of all real numbers. If $x$ is a real number then solving the equation for $y$ we see that $x$ is related to $y=\frac{5}{3}-\frac{2 x}{3}$. For instance $x=2$ is related to $y=\frac{1}{3}$. The range of $R$ is the set of all real numbers. If $y$ is a real number then solving the equation for $x$ we obtain that $x=\frac{5}{2}-\frac{3 y}{2}$ is related to $y$. For instance if $y=3$ then $x=-2$ is related to $y=3$.
Go back 2
3. Answer to Exercise 3. The domain and the range of $R$ is the set of all real numbers except for the number 0 . We explain how to find the domain only. If $x=0$ then for every value of $y$ we have $0 y=0$. It means that there is no value of y such that $0 y=1$. Thus the number 0 does not belong to the domain. If $x \neq 0$ then $x$ is related to $y=\frac{1}{x}$.
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4. Answer to Exercise 4. The domain of $R$ is the set of all real numbers because for every value of $x$ the number $x$ is related to $y=x^{2}-3$. The range of $R$ is the interval $[-3, x)$. If $x^{2} \geq 0$ then $x^{2}-3 \geq-3$ and $y \geq-3$. So we see that if $y<-3$ then there is no $x$ such that $y=x^{2}-3$. It means that $y$ does not belong to the range. If $y \geq-3$ then $y+3 \geq 0$ and the square root of $y+3$ is defined. So $x$ equal to $\sqrt{y+3}$ is related to $y$.
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5. Answer to Exercise 5. The domain is the set of all real numbers but, the number 2 because substitution $x=2$ leads to dividing by 0 . The range is the set of all real numbers but the number 1 because after solving the equation for $x$ we obtain $x=\frac{2 y}{(y-1)}$ which is undefined for $y=1$.
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6. Answer to Exercise 6. The domain is the interval [3, $X$ ) and the range is the set of all real numbers. Since for every value of $y$ we have $y^{2} \geq 0$ the value of $x$ needs to satisfy the inequality $x-3 \geq 0$ which gives $x \geq 3$.

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