

## 2.1. Polynomial functions.

In an earlier course, you learned about *polynomials* and *functions*.

In this lesson you will learn about *polynomial functions*.

### 2.1.1. DEFINITION.

A general polynomial function of  $x$  to the  $n^{\text{th}}$  degree is written:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

where  $n$  is a non-negative integer and  $a_n, a_{n-1}, \dots, a_2, a_1$ , and  $a_0$  are real numbers.

### 2.1.2. EXAMPLE. $f(x) = -6x^4 + 3x^3 + 8x^2 - x + 9$

- The *exponents* are: 4 (where  $n = 4$ ), 3 (where  $n-1 = 3$ ), 2 and 1. All exponents are non-negative integers.
- The exponent of greatest value,  $n = 4$ , is called the degree of the polynomial.
- The *coefficients* are:  $a_n = -6$ ,  $a_{n-1} = 3$ ,  $a_2 = 8$ ,  $a_1 = -1$ , and  $a_0 = 9$ .

The *leading coefficient* is  $a_n = -6$  and the constant term is  $a_0 = 9$ .

### 2.1.3. EXAMPLE.

Examples of Polynomial and Non-Polynomial Functions

Polynomial Function	Non-Polynomial Functions
$P(x) = 8$	$g(x) = 4x^2 - 11x^{-1}$
$f(x) = 3x$	$f(x) = \frac{x^2 - 1}{3x}$
$g(x) = 2x^7 - 145x^3 + 53$	$f(x) =  x $
$k(x) = x^4$	$f(x) = \sqrt{x^2 + 4}$

The notation used to represent a function is usually  $f(x)$  but other variables may be used.

Notice:

In the table above, functions have been named as  $f(x)$ ,  $g(x)$ ,  $P(x)$ ,  $h(x)$ , or  $k(x)$ . A polynomial function is often represented as  $P(x)$ .

Polynomial functions are classified according to their degree.

Degree	Polynomial functions	Classification
zero	<p>function:  <math>f(x) = a</math></p> <p>polynomial:  <math>f(x) = a_0</math></p>	<p><b>Constant Function:</b></p> <ul style="list-style-type: none"> <li>• Has <b>no</b> x-variable; therefore, the degree is zero.</li> <li>• Its graph is a horizontal line.</li> <li>• Has <b>no</b> x-intercepts.</li> <li>• Has a y-intercept of <math>(0, a)</math>.</li> </ul>
one	<p>function form:  <math>f(x) = ax + b</math> ; where  <math>a \neq 0</math></p> <p>polynomial form:  <math>f(x) = a_1x + a_0</math></p>	<p><b>Linear Function:</b></p> <ul style="list-style-type: none"> <li>• Its graph is a line with a slope of <math>a</math>.</li> <li>• Has one x-intercept <math>(-b/a, 0)</math>.</li> <li>• Has one y-intercept of <math>(0, b)</math>.</li> </ul>
two	<p>function form:  <math>f(x) = ax^2 + bx + c</math>  ; where <math>a \neq 0</math></p> <p>polynomial form:  <math>f(x) = a_2x^2 + a_1x + a_0</math></p>	<p><b>Quadratic Function:</b></p> <ul style="list-style-type: none"> <li>• Its graph is a parabola with vertex at <math>(-b/2a, f(-b/2a))</math>.</li> <li>• Has two x-intercepts, if the values of <math>x</math> are real,  <math>\left( \frac{-b + \sqrt{b^2 - 4ac}}{2a}, 0 \right)</math> and <math>\left( \frac{-b - \sqrt{b^2 - 4ac}}{2a}, 0 \right)</math></li> <li>• Has one y-intercept of <math>(0, c)</math>.</li> </ul>
three	<p>function form:  <math>f(x) = ax^3 + bx^2 + cx + d</math> ; where <math>a \neq 0</math></p> <p>polynomial form:  <math>f(x) = a_3x^3 + a_2x^2 + a_1x + a_0</math></p>	<p><b>Cubic Function:</b></p> <ul style="list-style-type: none"> <li>• <b>Its graph is a cubic curve.</b></li> <li>• <b>You will learn to find or approximate the x-intercepts.</b></li> <li>• <b>Has one y-intercept <math>(0, d)</math>.</b></li> </ul>
nth degree	<p>function form:  <math>f(x) = ax^n + bx^{n-1} + \dots + cx + d</math> ; where <math>a \neq 0</math></p> <p>polynomial form:  <math>f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0</math></p>	<p><b>Polynomial Function:</b></p> <ul style="list-style-type: none"> <li>• <b>Any other polynomial of degree greater than 3 is referred to as a polynomial of higher degree.</b></li> <li>• <b>Graph can be identified by its degree and its leading coefficient.</b></li> <li>• <b>The y-intercept is <math>(0, d)</math>.</b></li> </ul>