2.1. Polynomial functions.

In an earlier course, you learned about *polynomials* and *functions*.
In this lesson you will learn about *polynomial functions*.

2.1.1. DEFINITION.

A general polynomial function of x to the n^{th} *degree* is written:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$

where n is a non-negative integer and $a_n, a_{n-1}, \ldots a_2, a_1$, and a_0 are real numbers.

2.1.2. EXAMPLE. $f(x) = -6x^4 + 3x^3 + 8x^2 - x + 9$

- The *exponents* are: 4 (where $n = 4$), 3 (where n-1 = 3), 2 and 1. All exponents are non-negative integers.
- The exponent of greatest value, $n = 4$, is called the degree of the polynomial.
- The *coefficients* are: $a_n = -6$, $a_{n-1} = 3$, $a_2 = 8$, $a_1 = -1$, and $a_0 = 9$.

The *leading coefficient* is $a_n = -6$ and the constant term is $a_0 = 9$.

2.1.3. EXAMPLE.

Examples of Polynomial and Non-Polynomial Functions

Polynomial Function	Non-Polynomial Functions
$P(x) = 8$	$g(x) = 4x^2 - 11x^{-1}$
$f(x) = 3x$	$f(x) = \frac{x^2 - 1}{3x}$
$g(x) = 2x^7 - 145x^3 + 53$	$f(x) = \lvert x \rvert$
$k(x) = x^4$	$f(x) = \sqrt{x^2 + 4}$

The notation used to represent a function is usually $f(x)$ but other variables may be used.

Notice:

In the table above, functions have been named as $f(x)$, $g(x)$, $P(x)$, $h(x)$, or $k(x)$.
A polynomial function is often represented as $P(x)$.

Polynomial functions are classified according to their degree.

Degree	Polynomial functions	Classification
zero	function: $f(x) = a$ polynomial: $f(x) = a_0$	**Constant Function**: • Has **no** x-variable; therefore, the degree is zero. • Its graph is a horizontal line. • Has **no** x-intercepts. • Has a y-intercept of $(0,a)$.
one	function form: $f(x) =ax + b$; where $a \neq 0$ polynomial form: $f(x) = a_1x + a_0$	**Linear Function**: • Its graph is a line with a slope of a. • Has one x-intercept $(-b/a ,0)$. • Has one y-intercept of $(0,b)$.
two	function form: $f(x) = ax^2 + bx + c$; where $a \neq 0$ polynomial form: $f(x) = a_2x^2 + a_1x + a_0$	**Quadratic Function**: • Its graph is a parabola with vertex at $(-b/2a, f(-b/2a))$. • Has two x-intercepts, if the values of x are real, • $\left(\frac{-b+\sqrt{b^2-4ac}}{2a}, 0\right)$ and $\left(\frac{-b-\sqrt{b^2-4ac}}{2a}, 0\right)$ • Has one y-intercept of $(0,c)$.
three	function form: f(x) = ax3 + bx2 + cx + d ; where a ≠ 0 polynomial form: f(x) = a3x3+ a2x2 + a1x + a0	**Cubic Function:** • **Its graph is a cubic curve.** • **You will learn to find or approximate the x-intercepts.** • **Has one y-intercept (0, d).**
nth degree	function form: f(x) = axn + bxn-1 . . . + cx + d ; where a ≠ 0 polynomial form: f(x) = anxn+ an-1xn-1 + . . . + a2x2 + a1x + a0	**Polynomial Function:** • **Any other polynomial of degree greater than 3 is referred to as a polynomial of higher degree.** • **Graph can be identified by its degree and its leading coefficient.** • **The y-intercept is (0, d).**