### 2.4. Zeros of polynomial functions.

## Zeros of polynomial functions:

- are values of $x$ that satisfy a polynomial function $P(x)=0$.
- are determined by applying the Zero-Factor Property.
- are the same $x$ values expressed as solutions of the equation $P(x)=0$.
- are the same $x$ values expressed as $x$-intercepts of the graph of the function $P(x)$.
- are the same $x$ values expressed as $k$ in the factor $(x-k)$.

Example: If the zeros of polynomial $P(x)=3 x^{3}-x^{2}-8 x-4$ are $-1,-2 / 3$, and 2 , then

- $x=-1, x=-2 / 3$, and $x=2$ are solutions of the equation $P(x)=0$.
- $P(-1)=0, P(-2 / 3)=0, P(2)=0$ are values that satisfy the $P(x)=0$.
- $(-1,0) ;(-2 / 3,0)$; and $(2,0)$ are $x$-intercepts of the graph of the function $P(x)$.
$(x-(-1))(x-(-2 / 3))(x-(2))$ or in simplified form $(x+1)(3 x+2)(x-2)$ are factors of $P$.

In a previous lesson, we found zeros of polynomial functions by factoring.

## Let's review.

| STEPS TO FIND ALL ZEROS OF $\begin{gathered} P(x)=x^{4}-8 x^{2}+ \\ 16 \end{gathered}$ | COMMENTS |
| :---: | :---: |
| $\begin{aligned} & P(x)=x^{4}-8 x^{2}+ \\ & 16 \end{aligned}$ | If the degree of the polynomial is 4 , then there are at most 4 distinct zeros. |
| $P(x)=\left(x^{2}-4\right)\left(x^{2}-\right.$ <br> 4) or $P(x)=\left(x^{2}-\right.$ <br> 4) ${ }^{2}$ | Factor the perfect square trinomial. |
| $\begin{aligned} & P(x)=(x-2)(x+ \\ & 2)(x-2)(x+2) \end{aligned}$ | Factor the difference of two squares. |
| $\begin{array}{lc} (x-2)=0 ; & (x \\ +2)=0 ; & (x- \\ 2)=0 ; & (x+ \\ 2)=0 & \\ 2) \end{array}$ | Apply Zero-Factor Property -- 4 linear equations |
| $\begin{array}{rlrl}  & x= \\ 2 ; & x & =- \\ 2 & x= \end{array}$ | 4 distinct zeros, but only 2 zeros each with multiplicity of 2. |


| $2 ;$ | $x=$ |  |
| :--- | :--- | :--- |
| -2 |  |  |
| Zeros are <br> and <br> and |  |  |

## Remember! Some zeros are rational, irrational or complex.

Find the zeros of $P(x)=x^{3}+8$. You will be prompted by responding to the quiz questions.

So, the zeros of the polynomial $P(x)=x^{3}+8$ are $-2, x=1+i \sqrt{3}, x=1-i \sqrt{3}$.
Here is another example to find all zeros of a polynomial given one zero. Again you will be prompted by responding to the following quiz questions.

Let's Review.
$P(x)=x^{4}+10 x^{3}+27 x^{2}+10 x+26 ;$ given the zero of $i$

