

4.2. Exponential functions.

So far you have dealt with only **algebraic functions**, which included polynomial and rational functions. You will now study one type of nonalgebraic function called an **exponential function**.

Definition of Exponential Function:

The **exponential function f with base a** is denoted by $f(x) = a^x$ where $a > 0$, $a \neq 1$, and x is any real number.

Example: $f(x) = 2^x$ is an exponential function because $a = 2$ which verifies $a > 0$ and $a \neq 1$.

Note: a is the base and the base must be a positive real number with the exception of the number one.

Example: $f(x) = x^2$ is **not** an exponential function because the base must be a specified positive real number with the exception of one. This is an example of an algebraic function.

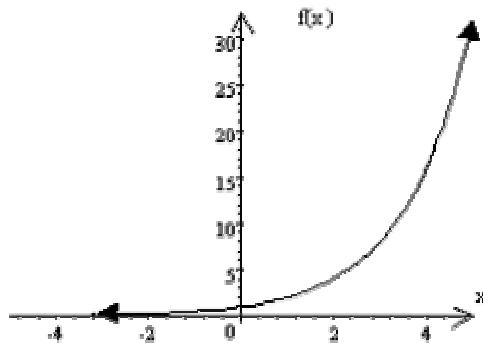
Note: The base $a = 1$ is excluded because it yields $f(x) = 1^x = 1$. This is a constant function, not an exponential function.

One can graph **exponential functions** by finding several ordered pairs (points) that belong to the function. Plotting these points (minimum of five) and connecting them with a smooth curve gives the graph.

Example: Graph the exponential function $f(x) = 2^x$.

Note: Begin by choosing random integer numbers for x . Calculate the value of $f(x)$ by substituting each chosen value of x .

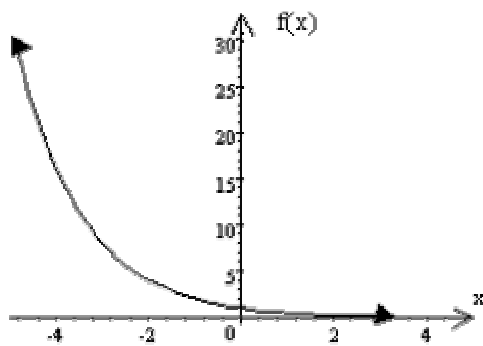
Ordered pairs of the graph of $f(x) = 2^x$ are: $(-3, 1/8)$, $(-2, 1/4)$, $(-1, 1/2)$, $(0, 1)$, $(1, 2)$, $(2, 4)$, and $(3, 8)$.



Note: As one moves from left to right, the graph is *increasing*.

Example: Graph the exponential function $f(x) = (1/2)^x$.

Ordered pairs of the graph are: $(-3, 8)$, $(-2, 4)$, $(-1, 2)$, $(0, 1)$, $(1, 1/2)$, $(2, 1/4)$, and $(3, 1/8)$.



Note: As one moves from left to right, the graph is *decreasing*.

Note: $f(x) = (1/2)^x$ can be written as $f(x) = 2^{-x}$ by using the properties of exponents discussed in lesson 1 of this module.

Based on the examples in the previous page, one can make the following generalizations about the **graphs of exponential functions** of the form $f(x) = a^x$.

1. The point $(0, 1)$ is on the graph.
2. If $a > 1$, then $f(x)$ is an increasing function. If $0 < a < 1$, then $f(x)$ is a decreasing function.
3. The x -axis is a horizontal asymptote.
4. The domain is the set of all real numbers, and the range is the set of positive real numbers.

Evaluating the Natural Exponential Function

Scientific Calculator:

Number:

$$e^2$$

Keystrokes:

$$2 \quad e^x \quad =$$

Display:

7.3890561

Number:

$$e^{-1}$$

Keystrokes:

$$1 \quad +/- \quad e^x \quad =$$

Display:

0.3678794

Graphing Calculator:

Number:

$$e^2$$

Keystrokes:

$$e^x \quad (-) \quad \text{Enter}$$

Display:

7.3890561

Number:

$$e^{-1}$$

Keystrokes:

e^x (-) 1 Enter

Display:

0.3678794

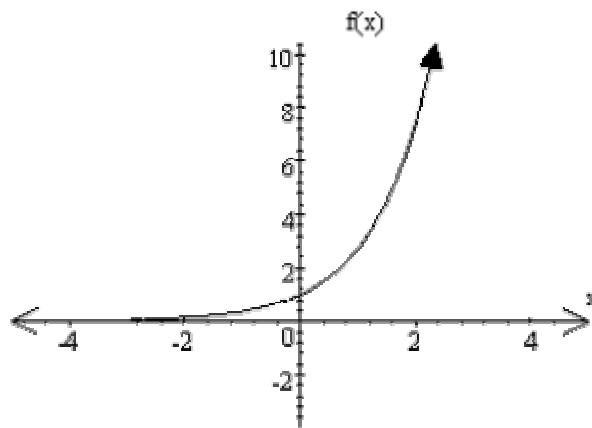
In many applications, the most convenient choice for a base is the irrational number $e \approx 2.71828\dots$ called the **natural base**.

The function $f(x) = e^x$ is called the **natural exponential function**.

Example: Graph the natural exponential function $f(x) = e^x$.

Note: Begin by choosing positive and negative integers for x as well as 0. Calculate $f(x)$ by substituting in the chosen x value.

Ordered pairs of the graph of $f(x) = e^x$ are: $(-3, e^{-3})$, $(-2, e^{-2})$, $(-1, e^{-1})$, $(0, 1)$, $(1, e)$, $(2, e^2)$, and $(3, e^3)$.



Note: The natural exponential function of the form $f(x) = e^x$ will also be an increasing or decreasing function.

The above graph is an example of an increasing function.