4.2. Exponential functions.

So far you have dealt with only algebraic functions, which included polynomial and rational functions. You will now study one type of nonalgebraic function called an exponential function.

Definition of Exponential Function:

> The exponential function f with base a is denoted by $f(x) = a^x$ where $a > 0$, $a \neq 1$, and x is any real number.

Example: $f(x) = 2^x$ is an exponential function because $a = 2$ which verifies $a > 0$ and $a \neq 1$.

Note: *a is the base and the base must be a positive real number with the exception of the number one.*

Example: $f(x) = x^2$ is **not** an exponential function because the base must be a specified positive real number with the exception of one. This is an example of an algebraic function.

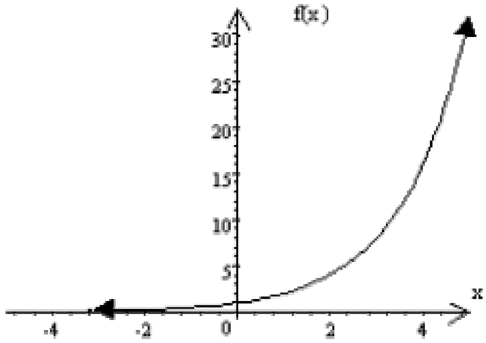
Note: *The base $a = 1$ is excluded because it yields $f(x) = 1^x = 1$. This is a constant function, not an exponential function.*

One can graph exponential functions by finding several ordered pairs (points) that belong to the function. Plotting these points (minimum of five) and connecting them with a smooth curve gives the graph.

Example: Graph the exponential function $f(x) = 2^x$.

Note: *Begin by choosing random integer numbers for x. Calculate the value of f(x) by substituting each chosen value of x.*

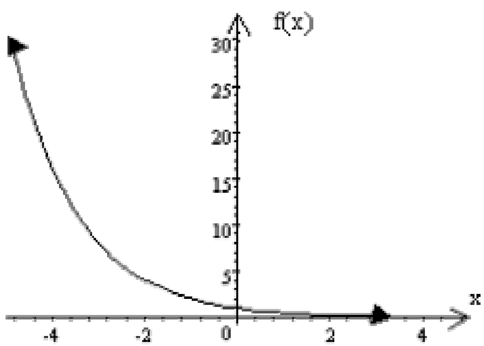
Ordered pairs of the graph of $f(x) = 2^x$ are: $(-3,1/8)$, $(-2,1/4)$, $(-1,1/2)$, $(0,1)$, $(1,2)$, $(2,4)$, and $(3,8)$.



Note: *As one moves from left to right, the graph is increasing.*

Example: Graph the exponential function $f(x) = (1/2)^x$.

Ordered pairs of the graph are: (–3,8), (–2,4), (–1,2), (0,1), (1,1/2), (2,1/4), and (3,1/8).



Note: *As one moves from left to right, the graph is decreasing.*

Note: *$f(x) = (1/2)^x$ can be written as $f(x) = 2^{-x}$ by using the properties of exponents discussed in lesson 1 of this module.*

Based on the examples in the previous page, one can make the following generalizations about the graphs of exponential functions of the form $f(x) = a^x$.

1. The point (0,1) is on the graph.
2. If $a > 1$, then f(x) is an increasing function. If $0 < a < 1$, then f(x) is a decreasing function.
3. The x-axis is a horizontal asymptote.
4. The domain is the set of all real numbers, and the range is the set of positive real numbers.

Evaluating the Natural Exponential Function

Scientific Calculator:

Number:

e^2

Keystrokes:

$2 \quad e^x \quad =$

Display:

7.3890561

Number:

e^{-1}

Keystrokes:

$1 \quad +/- \quad e^x \quad =$

Display:

0.3678794

Graphing Calculator:

Number:

e^2

Keystrokes:

$e^x \quad (-) \quad Enter$

Display:

7.3890561

Number:

e^{-1}

Keystrokes:

e^x $(-)$ 1 *Enter*

Display:

0.3678794

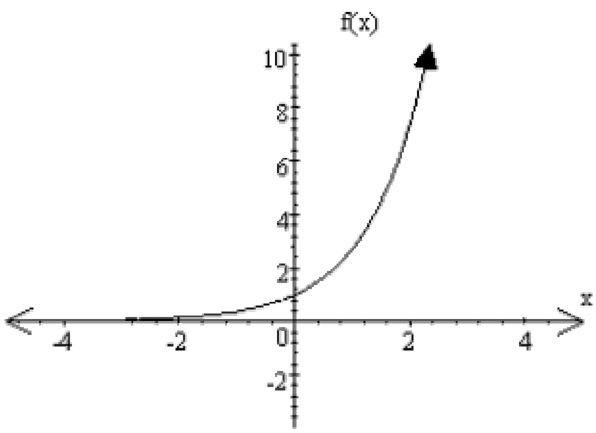
In many applications, the most convenient choice for a base is the irrational number e ≈ 2.71828… called the natural base.

The function $f(x) = e^x$ is called the natural exponential function.

Example: Graph the natural exponential function $f(x) = e^x$.

Note: *Begin by choosing positive and negative integers for x as well as 0. Calculate f(x) by substituting in the chosen x value.*

Ordered pairs of the graph of $f(x) = e^x$ are: $(-3,e^{-3})$, $(-2,e^{-2})$, $(-1,e^{-1})$, (0,1), (1,e), $(2,e^2)$, and $(3,e^3)$.



Note: *The natural exponential function of the form $f(x) = e^x$ will also be an increasing or decreasing function.*

The above graph is an example of an increasing function.