4.3. Logarithmic function.

Another type of nonalgebraic function is called a logarithmic function.

If a function has the property that no horizontal line crosses the graph of the function more than once, then the function must have an inverse. The graph of the exponential function of the form $f(x) = a^x$ or $y = a^x$ passes the "horizontal line test" and must therefore have an inverse. It just so happens that this inverse is called the logarithmic function with base a.

Definition of Logarithm:

For all positive numbers a, where $a \neq 1$, and all positive numbers x,

$y = \log_a x$ means the same as $x = a^y$.

Meaning of $\log_a x$:

A logarithm is an exponent.

$\log_a x$ is the exponent on the base a that yields the number x.

Note: *The equations $y = \log_a x$ and $x = a^y$ are equivalent. The first equation is in logarithmic form and the second is in exponential form.*

Examples: $\log_3 9 = 2$ because $3^2 = 9$.

$\log_4 1 = 0$ because $4^0 = 1$.

$\log_{10}(1/100) = -2$ because $10^{-2} = 1/10^2 = 1/100$.

Definition of Logarithmic Function:

If a and x are positive numbers, with $a \neq 1$, then $f(x) = \log_a x$ defines the logarithmic function with base a.

The logarithmic function with base 10 is called the common logarithmic function. On most calculators, this function is denoted by **log**.

Evaluating Common Logarithms on a Calculator

Scientific Calculator

Number:

$\log_{10}8$

Keystrokes:

8 log

Display:

0.903089987…

Number:

$3 \log_{10}4.5$

Keystrokes:

4.5 log $\times$ 3 =

Display:

1.959637541…

Number:

$\log_{10}(-7)$

Keystrokes:

7 $\pm$ log

Display:

ERROR

Note: *Logarithms of negative numbers do not exist, and can not be calculated. However, a logarithm can have a negative value.*

Graphing Calculator

Number:

$\log_{10}8$

Keystrokes:

$\log \quad 8 \quad \mathit{Enter}$

Display:

0.903089987…

Number:

$3\ \log_{10} 4.5$

Keystrokes:

$3 \quad \times \quad \log \quad 4.5 \quad \mathit{Enter}$

Display:

1.959637541…

Number:

$\log_{10}(-7)$

Keystrokes:

$\log \quad (-) \quad 7 \quad \mathit{Enter}$

Display:

ERROR

The logarithmic function $f(x) = \log_e x$ with base *e* (*e* = *2.718281828…*) is called the natural logarithmic function, and is denoted by the symbol **ln x**.

Evaluating Natural Logarithms on a Calculator

Scientific Calculator

Number:

ln 3

Keystrokes:

3 ln

Display:

1.098612289…

Number:

ln e^2

Keystrokes:

2 e^x ln

Display:

−1.2039728…

Number:

Ln (−4)

Keystrokes:

4 ± ln

Display:

Error

Graphing Calculator

Number:

ln 3

Keystrokes:

ln 3 *Enter*

Display:

1.098612289…

Number:

$\ln e^2$

Keystrokes:

ln e^x 2 *Enter*

Display:

−1.2039728…

Number:

Ln (−4)

Keystrokes:

ln (−) 4 *Enter*

Display:

Error

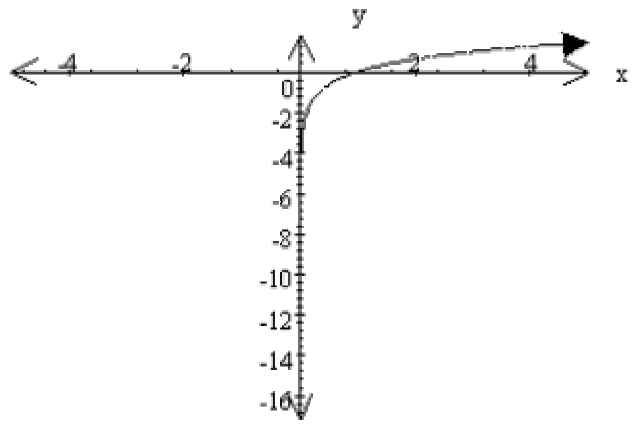
To graph a logarithmic function it is helpful to write it in exponential form first. Then plot selected ordered pairs to determine the graph.

Example: Graph the logarithmic function $f(x) = \log_3 x$.

Note that f(x) means the same as y.

First begin by rewriting the equation as $x = 3^y$.

Selected ordered pairs belonging to the graph of this equation are: (1/9,− 2), (1/3,− 1), (1,0), (3,1), and (9,2).



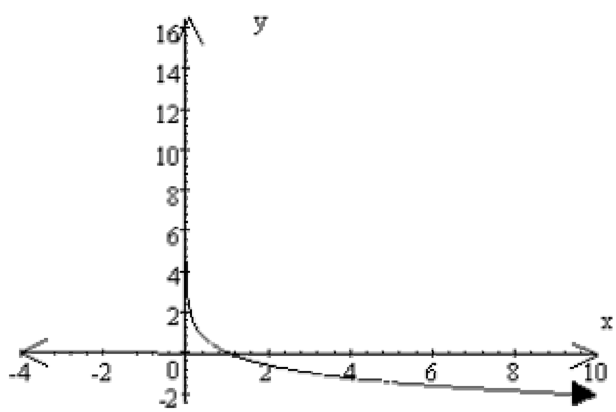
Note: *As one moves from left to right, the graph is increasing.*

Example: Graph the logarithmic function $f(x) = \log_{1/3}x$.

Rewrite the equation as $x = (1/3)^y$ (Remember that $f(x) = y$).

The equation could also be rewritten as $x = 3^{-y}$.

Selected ordered pairs belonging to the graph of this equation are: (9,– 2), (3,– 1), (1,0), (1/3,1), and (1/9,2).



Note: *As one moves from left to right, the graph is decreasing.*

Based on the examples on the previous page, one can make the following generalizations about the graphs of logarithmic functions of the form $f(x) = \log_a x$.

1. The graph will always contain the point (1,0).

2. If a > 1, then the graph will *rise (increase)* from left to right.

   If 0 < a < 1, then the graph will *fall (decrease)* from left to right.

3. The y-axis is a vertical asymptote. That is, the graph gets very close but will not cross.
4. The domain is the set of positive real numbers and the range is the set of all real numbers.

Note: *When comparing these generalizations with those of the exponential function discussed in lesson 3 page 2 of this module, one will note the inverse that is occurring.*