4.4. Properties of logarithms.

Logarithms have been used as an aid to numerical calculations for several hundred years. The widespread use of calculators today has made the use of logarithms for calculation obsolete. However, logarithms are still useful in applications and in further work in mathematics. There are several properties (rules) of logarithms that are useful and are illustrated in this lesson.

Properties (Rules) of Logarithms: If $x > 0$, $y > 0$, $b > 0$, $b \neq 1$, and if r is any real number, then

1. **Product Rule:** $\log_b(xy) = \log_b x + \log_b y$

   Example: $\log_3(4x) = \log_3 4 + \log_3 x$

   Note: *The logarithm of a product is the sum of the logarithms of the factors.*

2. **Quotient Rule:** $\log_b(x/y) = \log_b x - \log_b y$

   Example: $\log_4(3/5) = \log_4 3 - \log_4 5$

   Note: *The logarithm of a quotient is the difference between the logarithm of the numerator and the logarithm of the denominator.*

3. **Power Rule:** $\log_b(x^r) = r(\log_b x)$

   Example: $\log_5(2^3) = 3(\log_5 2)$

   Note: *The logarithm of a number to a power equals the exponent times the logarithm of the number.*

4. **Special Rules:**

   a. $\log_b b = 1$

   Example: $\log_5 5 = 1$

   b. $\log_b 1 = 0$

   Example: $\log_3 1 = 0$

c. $\log_b b^x = x$

Example: $\log_2 2^7 = 7$

d. $b^{\log_b x} = x$

Example: $3^{\log_3 7} = 7$

5. Change of Base Rule: If $a > 0$, $a \neq 1$, $x > 0$, $b = 10$, then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Example:

$$\log_5 7 = \frac{\log_{10} 7}{\log_{10} 5} = 1.209061955 \ \ldots$$

Most of the properties of logarithms illustrated on the previous page can also be applied to the natural logarithm (ln).

If $x > 0$, $y > 0$, and if r is any real number, then

1. Product Rule: ln(xy) = ln x + ln y

Example: ln(4y) = ln 4 + ln y

2. Quotient Rule: ln(x/y) = ln x – ln y

Example: ln(5/6) = ln 5 – ln 6

3. Power Rule: $\ln(x^r) = r(\ln x)$

Example: $\ln(2^3) = 3(\ln 2)$

4. Special Rules:

a. $\ln e = 1$

b. $\ln 1 = 0$

c. $\ln e^x = x$