

4.5. Applications of exponential and logarithmic functions.

Exponential and logarithmic functions are used to model several real-life situations.

Some of them are:

- **Radioactive Decay:** $A = A_0(2^{-t/h})$ where A_0 is the amount present at $t = 0$ and h is the materials half-life.
- **Compound Interest:** $A = A_0(1 + r/k)^{kt}$ where A represents the amount in the account after t years, with interest paid k times a year at an annual rate of r percent on an initial deposit A_0 .
- **Continuous Compound Interest:** $A = A_0e^{rt}$ where A_0 is the initial deposit, r is the rate of percent, and t is the time in years.
- **Population Growth:** $P = P_0e^{kt}$ where P_0 is the population at $t = 0$, and $k = B - D$. Population growth assumes a constant birth rate B and a constant death rate D . If t is measured in years, then k is called the annual growth rate.
- **pH of a Solution:** $\text{pH} = -\log[\text{H}^+]$ where $[\text{H}^+]$ is the hydrogen-ion concentration in grams-ions per liter.
- **Decibel Voltage Gain:** $D = 20 \log(E_0/E_I)$ where E_0 is the output voltage of a device, and E_I is the input voltage.
- **Intensity of Earthquakes Measured on Richter Scale:** $R = \log(A/P)$ where A is the amplitude of the tremor measured in micrometers and P is the period of the tremor (time of one oscillation of the earth's surface measured in seconds).
- **Weber-Fechner Law:** $L = k \ln(I)$ where L is the apparent loudness of a sound, and I is the actual intensity of a sound.

- **Depreciation:**

$$n = \frac{\log V - \log C}{\log \left(1 - \frac{2}{N}\right)}$$

where N is the life expectancy, C is the cost in dollars, V is the value in dollars to which the object depreciates, and n is the age of the object.