5.3. Matrices and systems of linear equations

Basic properties of matrices

The numbers $1, 2, 3, 4$ could be organized in the following rectangular array which is called a matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

This matrix has two rows which run horizontally

$$(1\ 2) \text{ and } (3\ 4).$$

It has two columns which run vertically

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Each number in the matrix is called an *entry* and it has its location which is described by specifying the row and the column to which the number belongs. For instance, the number 2 is located in the first row and the second column. The number 3 is located in the second row and the first column. The way in which we locate the numbers is essential. The matrix

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

consists of the same numbers but is different from the first one. We say that two matrices are equal if they consist of the same numbers and the numbers are located in the same way.

Matrices can have more rows and columns

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

and different number of rows than columns

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

For each matrix we define its *dimension* which tells how many rows and how many column the matrix has. A matrix with 3 rows and 5 columns

has dimension 3×5 (read "3 by 5"). In general, matrix with k rows and m columns has dimension $k \times m$.

Sometimes the last column is separated with a vertical line

$$\begin{pmatrix} 1 & 1 & | & 1 \\ 2 & 2 & | & 3 \end{pmatrix}$$

and we call such a matrix an *augmented* matrix.

Associating matrices with systems of linear equations

There are two matrices that we associate with a system of linear equations: coefficient matrix and augmented matrix. The first one consists of numbers in the front of the variables located in this way that if the numbers appears in the third equation in the front of the second variable we write it in the third row and the second column. The augmented matrix has one more column which consists of the constant numbers on the right sides of the equations and usually is separated with a vertical line.

For the system

$$\begin{array}{rrrrr} 2x & + & y & = & 1 \\ 3x & - & 5y & = & 2 \end{array}$$

the coefficient matrix is

$$\begin{pmatrix} 2 & 1 \\ 3 & -5 \end{pmatrix}$$

and the augmented matrix is

$$\begin{pmatrix} 2 & 1 & | & 1 \\ 3 & -5 & | & 2 \end{pmatrix}$$

5.3.1. EXAMPLE.
The augmented matrix of the system

$$\begin{array}{rrrrrrr} 3x & - & y & + & 13z & = & 21 \\ 2x & + & 4y & & & = & 2 \\ 3x & + & 3y & + & 3z & = & 15 \end{array}$$

is

$$\begin{pmatrix} 3 & -1 & 13 & | & 21 \\ 2 & 4 & 0 & | & 2 \\ 3 & 3 & 3 & | & 15 \end{pmatrix}.$$

Let us notice that the number located in the second row and the third column equals 0. It because the second equation has no term with the variable z.