

6.4. Applications of sequences.

There are many **applications of sequences**. To solve problems involving sequences, it is a good strategy to list the first few terms, and look for a pattern that aids in obtaining the general term. When the general term is found, then one can find any term in the sequence without writing all the preceding terms.

Sequences are useful in our daily lives as well as in higher mathematics. For example, the interest portion of monthly payments made to pay off an automobile or home loan, and the list of maximum daily temperatures in one area for a month are sequences.

Example: A child building a tower with blocks uses 15 for the bottom row. Each row has 2 fewer blocks than the previous row. Suppose that there are 8 rows in the tower.

(a) How many blocks are used for the top row?

Note: The number of blocks in each row forms an **arithmetic sequence** with $a_1 = 15$ and $d = -2$. Find a_n for $n = 8$ by using the formula $a_n = a_1 + (n - 1)d$.

$$a_8 = 15 + (8 - 1)(-2)$$

$$a_8 = 1.$$

There is just one block in the top row.

(b) What is the total number of blocks in the tower?

Note: Here we must find the **sum of the terms of the arithmetic sequence** formed with $a_1 = 15$, $n = 8$, and $a_8 = 1$ by using the formula $S_n = n/2(a_1 + a_n)$.

$$S_8 = 8/2(15 + 1)$$

$$S_8 = 4(16)$$

$$S_8 = 64.$$

There are 64 blocks in the tower.

Example: An insect population is growing in such a way that each new generation is 1.5 times as large as the previous generation. Suppose there are 100 insects in the first generation.

(a) How many will there be in the fifth generation?

Note: The population can be written as a **geometric sequence** with a_1 as the first-generation population, a_2 as the second-generation population, and so on. Then the fifth-

generation population will be a_5 . Find a_5 by using the formula $a_n = a_1 r^{n-1}$ with $n = 5$, $r = 1.5$, and $a_1 = 100$.

$$a_5 = 100(1.5)^{5-1}$$

$$a_5 = 100(1.5)^4$$

$$a_5 = 506.25.$$

In the fifth-generation, the population will number about 506 insects.

(b) What will be the total number of insects in the five generations?

Note: Find the sum of the first five terms using the formula for the *sum of the first n terms of a geometric sequence*.

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_5 = \frac{100(1 - (1.5)^5)}{1 - 1.5}$$

$$S_5 = \frac{100(1 - 7.59375)}{-0.5}$$

$$S_5 = 1318.75$$

The total population for the five generations will be about 1319 insects.

Another important application of sequences is their use in compound interest and simple interest.

Example: If Linda deposits \$1300 in a bank at 7% interest compounded annually, how much will be in the bank 17 years later?

Note: Use the compound interest formula, $A = P(1 + r/k)^{kt}$ with $P =$ principle, $t =$ time in years, $r =$ annual rate, and $k =$ number of periods per year.

$$A = 1300(1 + 0.07/1)^{1 \cdot 17}$$

$$A = 1300(1 + 0.07)^{17}$$

$$A = 4106.46.$$

The account will contain \$4,106.46.

Example: Find the accumulated value of \$15,000 at 5% per year for 18 years using simple interest.

Note: Use the simple interest formula, $I = Prt$ with $I =$ total interest, $P =$ principle, $r =$ annual rate, and $t =$ time in years.

$$I = 15,000(0.05)(18)$$

$$I = 13,500.$$

A total of \$13,500 in interest will be earned.

Hence, the accumulated value in the account will be $13500 + 15000 = \$28,500$.